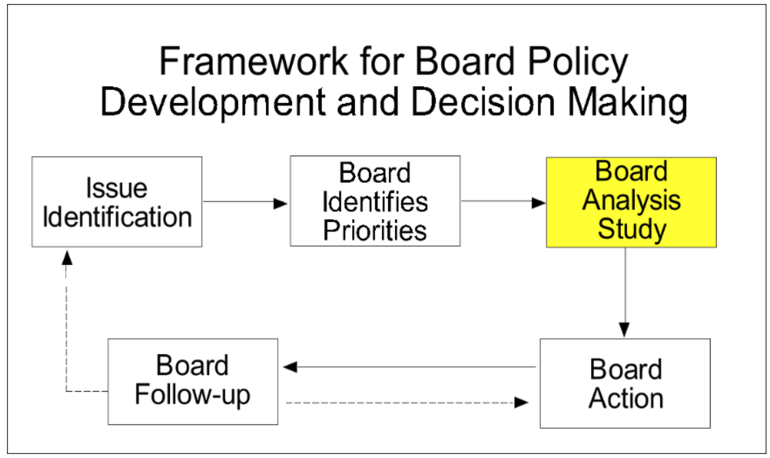
Iowa State Board of Education

Executive Summary

March 21, 2024



Agenda Item: Revision of Iowa Academic Standards for Mathematics – First Read

State Board Priority: Eliminating Achievement and Opportunity Gaps

State Board Role/Authority: Iowa Code section 256.7, subsection 26 and 28 direct the State Board to adopt content standards for mathematics education.

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Attachment(s): One

Recommendation: It is recommended that the State Board hear and discuss this information.

Background: The State Board adopted standards for mathematics in September 2010.

IOWA | Department of Education

IOWA ACADEMIC STANDARDS FOR

Mathematics

May 2024

Contents

Introduction

The Iowa Academic Standards for Mathematics serve as a compass, guiding educators and students through the rich landscape of mathematical learning. These standards meticulously outline the knowledge, skills, and understandings that students should know and be able to do because of their mathematical instruction and experiences. Like any robust standards, these standards rigorously demand a delicate balance of conceptual understanding, procedural fluency, and real-world application. The standards represent a pinnacle of achievement in mathematics, equipping students with the proficiency needed for seamless transitions into post-secondary education and the workforce.

While these standards provide a vital roadmap, their success relies on the use of high-quality instructional materials. Iowa educators are encouraged to use high-quality instructional resources meticulously aligned with the Iowa Academic Standards for Mathematics, ensuring that all students have equitable access to enriching learning experiences. High-quality instructional materials are pivotal in nurturing critical thinking, problem solving, and mathematical reasoning among learners of all backgrounds.

The evolution of these standards reflects a commitment to continuous improvement, an ongoing effort to meet the evolving needs of students and society. The revised standards represent not merely a reiteration of past practices but an opportunity for growth and refinement. The structure of the standards can be thought of as a DNA strand containing smaller components and does not work well in isolated parts when taken apart. The standards are not a checklist. There is a structure to the standards, and the standards are the structure.

Central to these standards is a mathematical narrative that unfolds across grade levels, like episodes in a television series. Just as watching episodes out of order disrupts the narrative’s coherence, the strategic sequencing and connection of mathematical concepts are essential for meaningful learning. Each lesson, unit, and grade level builds upon the previous, fostering a continuum of mathematical understanding and competence. Moreover, adherence to the standards for mathematical practices empowers students to become adept problem solvers and effective communicators of mathematical reasoning.

There are four main components to the Iowa Academic Standards for Mathematics.

- Content Standards
- Standards for Mathematical Practice
- The Three Shifts in Mathematics Instruction
- Effective Teaching Practices

Together, these components form the cornerstone of mathematics education in Iowa, embodying a commitment to excellence and opportunity for all learners

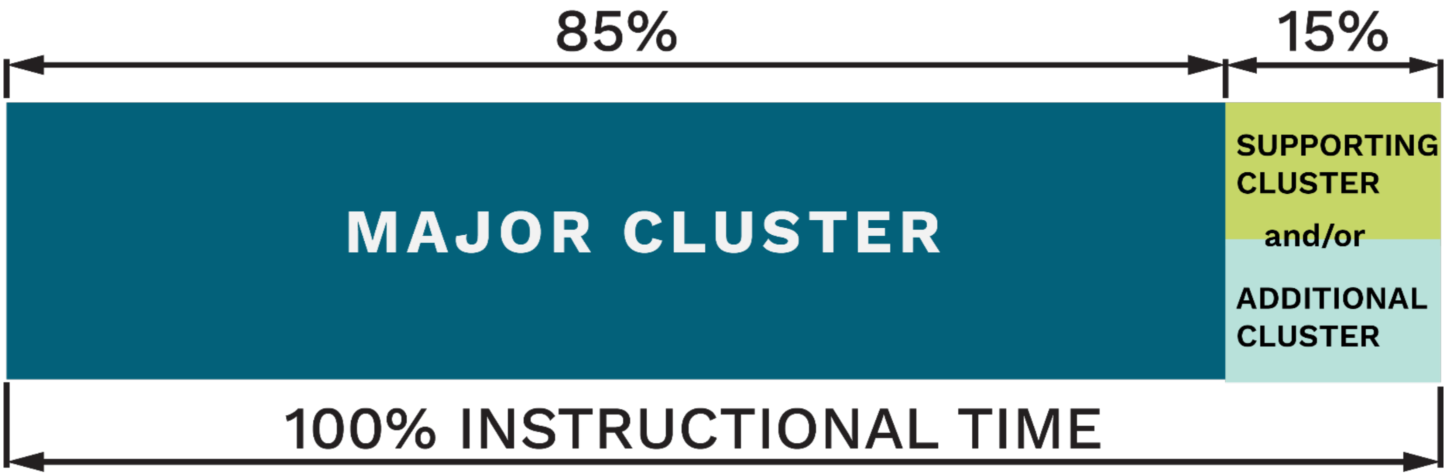
The Three Shifts in Mathematics Instruction

The Shifts serve as a framework that outlines how these standards elevate expectations across various aspects of a students’ educational journey, encompassing instructional materials, classroom practice, and assessment. They demonstrate how college and career-ready standards drive transformative changes in the classroom, better-preparing students for opportunities after high school.

1. The Shift of Focus

The first shift requires prioritizing the Major Work of each grade level. Rather than trying to cover topics superficially, the Iowa Academic Standards for Mathematics urge us to significantly narrow and deepen the focus of time and energy in the mathematics classroom. We delve deeply into the Major Work of each grade to ensure students establish robust foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply mathematical concepts to real-world problem-solving scenarios.

A greater emphasis is placed on clusters, which consist of groups of related standards. Proficiency at the cluster level in each grade is essential for students to thrive in later grades. Clusters encompass Major, Supporting, or Additional Work of the grade, with the standards beneath the cluster heading offering detailed insights into the mathematics within the cluster. Proficiency at the cluster level serves as the benchmark for proficiency.



Major Work: Educators should allocate most of their classroom time to engaging with the "Major Work of the Grade." At least 65% and up to 85% of class time should be dedicated to the Major Work, as it lays the groundwork for high school algebra readiness.

Grade Band/Level	Highlights of Major Work in K-12
K–2	Addition and subtraction—concepts, skills, and problem solving; place value
3–5	Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving
6	Ratios and proportional relationships; early expressions and equations
7	Ratios and proportional relationships; arithmetic of rational numbers
8	Linear equations and linear functions
Algebra 1	Modeling with linear, quadratic, and exponential equations and functions
Geometry	Modeling with congruence and similarity
Algebra 2	Modeling with polynomial and rational equations and functions

Attending to the Major Work in K–8 is crucial because it sets the foundation for high school algebra readiness. Not all content in each grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time they take to become proficient, and their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

The Major Work in kindergarten is all of the major clusters in kindergarten. Similarly, the major clusters in each grade level are the clusters that tie directly to the K–8 arithmetic to algebra progression arrow for high school readiness. The Major Work in high school consists of the major clusters in the courses that tie directly to college and career readiness.

K — Kindergarten

- Know number names and the count sequence forward.
- Count to tell the number of objects.
- Compare numbers.
- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
- Work with numbers 11-19 to gain foundations for place value.

1 — First Grade

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.
- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.
- Measure lengths indirectly and by iterating length units.

2 — Second Grade

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.
- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.

3 — Third Grade

- Represent and solve problems involving multiplication and division.
- Use properties of operations and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.
- Understand fractions as numbers.
- Solve problems with time, money and measured quantities.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

4 — Fourth Grade

- Use the four operations with whole numbers to solve problems.
- Generalize place value understanding for multi-digit whole numbers up to 1,000,000.
- Calculate with multi-digit numbers.
- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions for tenths and hundredths.

5 — Fifth Grade

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.
- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and addition.

6 — Sixth Grade

- Apply ratio concepts and use ratio reasoning to solve problems.
- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Apply and extend previous understandings of numbers to the system of rational numbers.
- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

7 — Seventh Grade

- Analyze proportional relationships and use them to solve real-world and mathematical problems.
- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

8 — Eighth Grade

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.
- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.
- Demonstrate congruence and similarity using physical models, patty paper or geometry software.
- Explain and apply the Pythagorean Theorem.

Supporting Work and **Additional Work** complement the Major Work of each grade level. Stating that Major Work has a greater emphasis does not imply that the Supporting or Additional Work of the Grade can be safely neglected in instruction. Supporting and Additional Work does not mean optional. Neglecting Supporting and Additional Work can lead to gaps in student skill and understanding, potentially leaving students ill-equipped for the rigors of later grades.

Supporting Work closely aligns with the Major Work in K–8, reinforcing and enriching core concepts. Meanwhile, Additional Work indirectly supports the Major Work in K–8 and sets the stage for success in high school. A comparable relationship exists in high school regarding post-secondary readiness and success.

Additional or Supporting Clusters are included to enhance the grade-level focus. For instance, instead of data displays as an end in themselves, they are opportunities to solve grade-level word problems. Additional does not mean option. “Additional” identifies which clusters have connections farther away from the Major Clusters than Supporting Work.

2. The Shift of Coherence

The second shift requires coherence within and across grade levels to ensure instruction follows a logical mathematical progression. Iowa Academic Standards for Mathematics are connected and coherent progressions from one grade to the next. Learning is thoughtfully interconnected across grades, allowing students to build upon foundations established in previous years. Each standard serves as an extension of prior learning rather than a standalone event.

The Coherence Map visually represents the connections between Standards for Mathematics.

The Progression Documents for Mathematics explain mathematical topics’ learning trajectories and conceptual development within and across grades. They are vital when considering differentiation, intervention, and integration of technology.

3. The Shift of Rigor

The third shift clarifies the aspects of rigor needed to work with mathematical concepts. There are three aspects of rigor.

a. **Conceptual understanding:** The standards call for a conceptual understanding of key concepts. Students must be able to access concepts from several perspectives to see mathematics as more than a set of mnemonics or discrete procedures.
b. **Procedural skill and fluency:** Fluency is the ability to apply procedures efficiently, flexibly, and accurately, including fact, computational, and procedural fluency. Critical end-of-grade-level standards are identified in grades K–8 where fluency should be expected by the end of the grade.
c. **Application:** Students use math flexibly for applications in problem solving in real-world contexts. In content areas outside of math, particularly science, students can use math to make meaning of and access content.

High-quality mathematics instruction should include all three aspects of rigor with equal intensity. Educators can visualize the three aspects of rigor as a three-legged stool. If one area of rigor is ignored or receives less intensity, it will fall over, thus not providing a stable foundation for students. Each standard aligns with one or more components of rigor.

The language of the standards reflects the aspects of rigor. Conceptual understanding standards often use words like understand, recognize, or interpret; procedural skill and fluency standards use words like fluently, find, or solve; and application standards typically use word problems or real-world problems.

The standards identify the aspects of rigor that educators should emphasize. The aspect of rigor identification does not imply that those are the sole aspects, but they are listed to help educators make instructional decisions. In the context of a shift, rigor does not mean harder. (Achieve the Core, 2024)

Content Standards

The mathematical content standards are the individual skills and competencies that students should know and be able to do. Grade levels consist of domains. Domains have clusters; clusters have individual standards. Educators must attend to cluster-level proficiency as it is the roadmap to the Work of the Grade and crucial to student success.

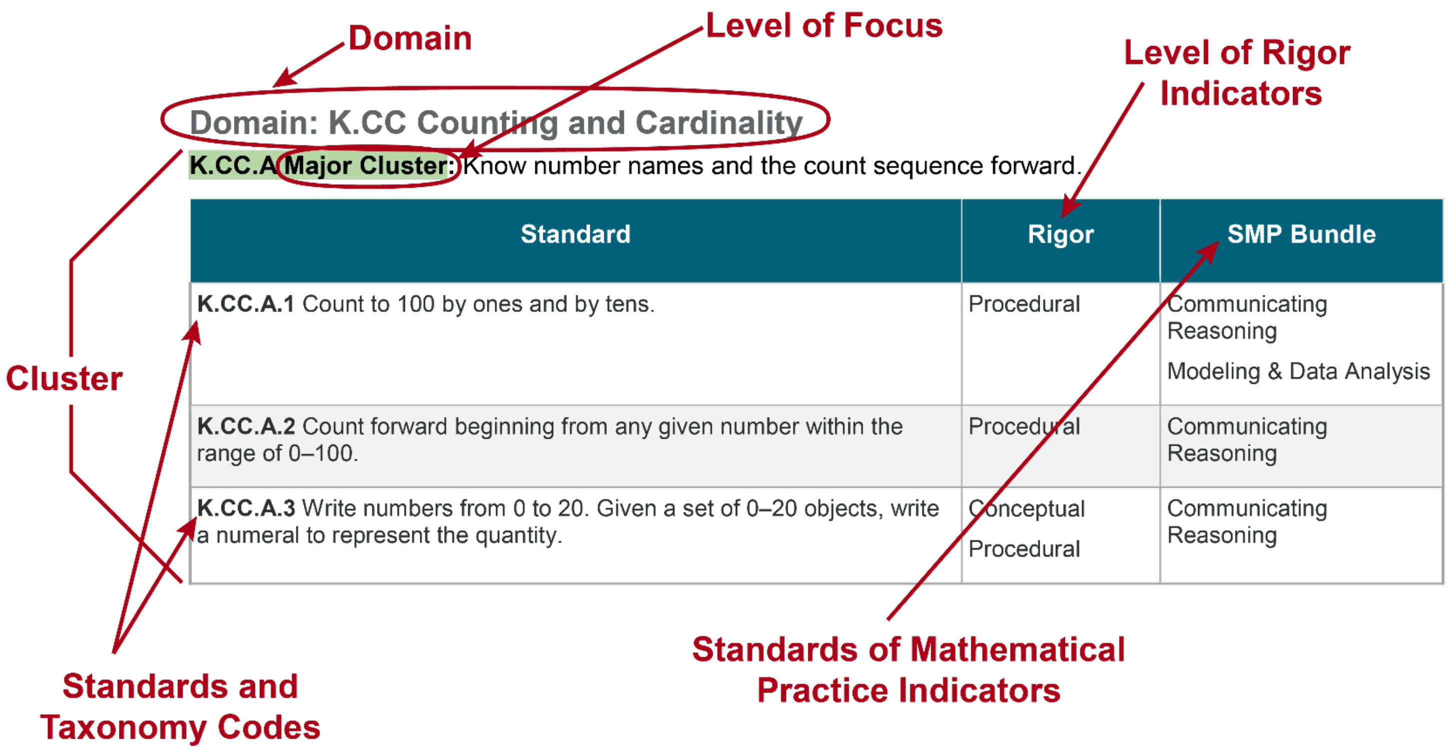
By “attending to” the cluster-level proficiency, educators can more deeply understand the context in which they are working and tailor their instruction. While each domain has its own knowledge and skills, we expect proficiency at the cluster level. The standards within a cluster illustrate the mathematics contained in the cluster proficiency as related aspects. Educators should take great care not to cause atomization. Atomization occurs when the procedures and skills are whittled down so small making them useless. Educators should preserve the interconnectedness and relationships of mathematics.

How to Read the Standards

The standards are designed to offer a meaningful way to interpret what students should know and be able to do at each grade level. We use the items below to help clarify the information in the Standards document.

- **Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related**.**
- **Clusters** are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
- **Standards** define what students should understand and be able to do.
- **Level of Focus** identifies the content where the **majority** of the instructional time may be needed for all students to achieve proficiency. The three levels of focus are color coded as Major Clusters, Supporting Clusters, or Additional Clusters.
- **Aspects of Rigor** identifies which of the aspects of rigor applies to the standards, for example, conceptual understanding, procedural fluency and application.
- **Standards for Mathematical Practice Indicators** identifies which bundle of the Standards for Mathematical Practices need to be emphasized, for example, Problem Solving, Communicating Reasoning, and Mathematical Modeling and Data Analysis.
- **Taxonomy Codes** are unique, alpha numeric identifiers for each standard. For example, **1.OA.A.1** can be interpreted as: First Grade. Operations and Algebraic Thinking. Cluster A. Standard number 1. Standards unique to Iowa are coded with IA in the taxonomy.

When a standard includes a “for example” in italics, it is an illustration and not part of the standards.



Standards for Mathematical Practices

The Standards for Mathematical Practice (SMP) describe how students should engage with the subject matter as they grow in mathematical maturity and expertise throughout elementary, middle, and high school. Equally, the SMPs are varieties of expertise specific to mathematics; they reflect the discipline's habits of mind, ways of knowing, and intellectual virtues. These practices rest on important "processes and proficiencies" in mathematics education.

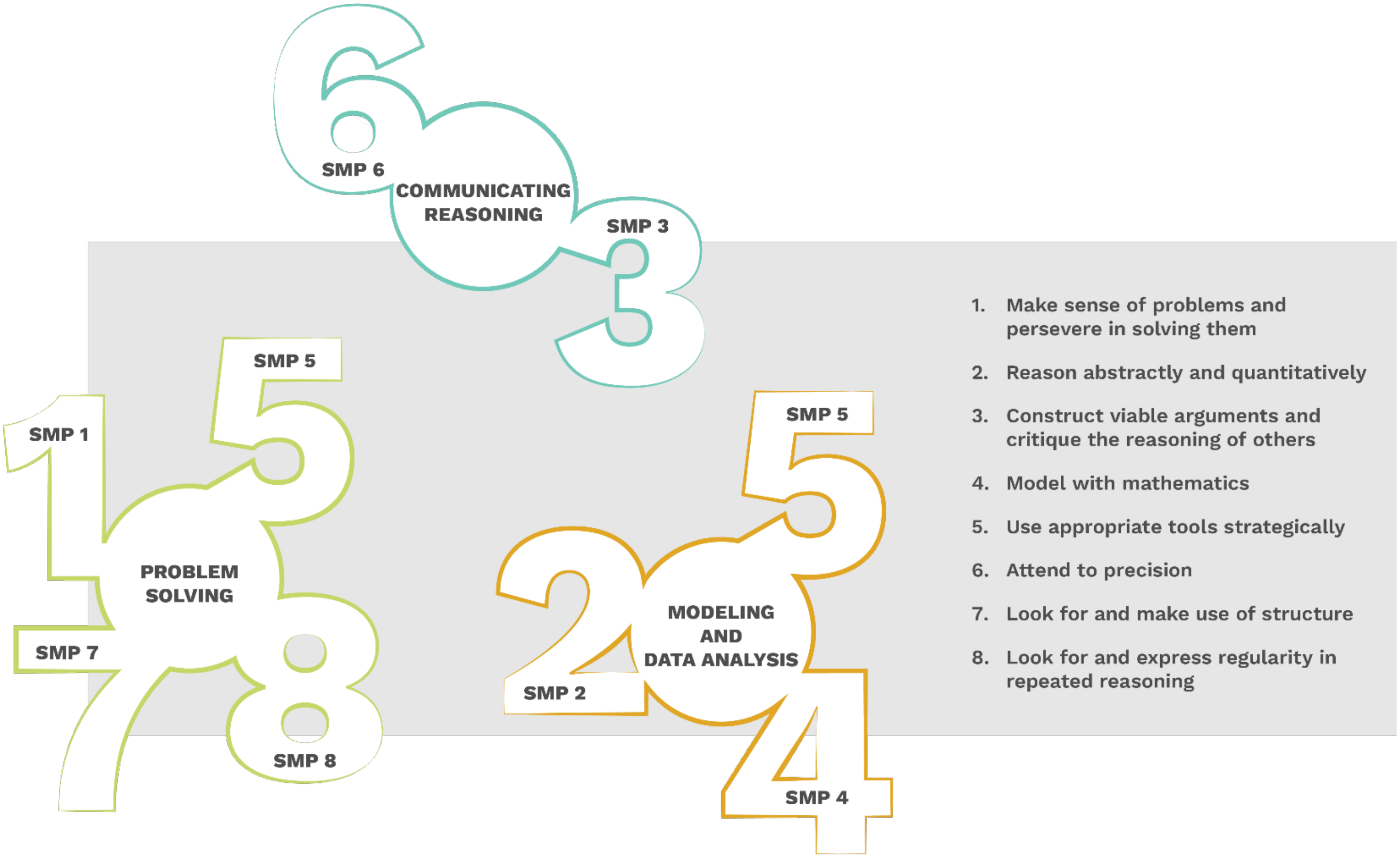
There are eight Standards for Mathematical Practices.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The SMPs are defined in grade bands with grade specific examples for grades K–5, grades 6–8, and high school. The grade band descriptors can be found at the beginning of each grade band within the body of the standards.

SMP Bundling: The eight practices can be bundled into three larger categories to support:

A. Problem Solving (SMPs 1,5,7,8)
B. Communicating Reasoning (SMPs 3,6)
C. Mathematical Modeling and Data Analysis (SMPs 2,4,5)



This bundling can help to focus the practices in instruction. It is the minimum bar for all students. Bundles are listed after the standards to help educators in emphasizing the most important ones with the corresponding standards. The bundle is not to limit educators but to assist with decision-making strategies.

The **Problem-Solving bundle** contains SMPs 1,5,7, and 8. In this bundle, students solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Problem Solving characteristics include:

1. Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace.
2. Select and use appropriate tools strategically.
3. Interpret results in the context of a situation.
4. Identify important quantities in a practical situation and map the relationships (for example, using diagrams, two-way tables, graphs, flowcharts, or formulas).

The **Communicating Reasoning bundle** contains SMPs 3 and 6. In this bundle, students clearly and precisely construct viable arguments to support their reasoning and critique the reasoning of others.

Communicating Reasoning characteristics include:

1. Test propositions or conjectures with specific examples.
2. Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.
3. State logical assumptions being used.
4. Use the technique of breaking an argument into cases.

5. Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is.
6. Base arguments on concrete referents such as objects, drawings, diagrams, and actions.
7. At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.)

The **Modeling and Data Analysis bundle** contains SMPs 2,4 and 5. In this bundle, students analyze complex, real-world scenarios and construct and use mathematical models to interpret and solve problems.

Modeling and Data Analysis characteristics include:

1. Apply mathematics to solve problems arising in everyday life, society, and the workplace.
2. Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.
3. State logical assumptions being used.
4. Interpret results in the context of a situation.
5. Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

(adapted from Smarter Balance Content Specifications, 2015)

Effective Teaching Practices

In the past twenty-five years, we have learned that standards alone—regardless of their origins, authorship, or development process—will not achieve the goal of fostering high levels of mathematical understanding among all students. In other words, more than just standards are needed. The National Council of Teachers of Mathematics (NCTM) has formulated *Principles to Actions: Ensuring Mathematical Success for All*, which outlines specific actions teachers and stakeholders must take to realize our shared objective of ensuring mathematical success for all. These actions encompass eight Effective Teaching Practices.

1. **Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. **Implement tasks that promote reasoning and problem-solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
3. **Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
4. **Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. **Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense-making about important mathematical ideas and relationships.
6. **Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
7. **Support productive struggle in learning mathematics**. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and support to engage in productive struggle as they grapple with mathematical ideas and relationships.
8. **Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

(From Principles to Actions, 2014)

Students develop deep conceptual understanding and skill proficiency by using rich mathematical tasks. Rich mathematical tasks involve teaching through problem solving with problem-based instructional tasks. It also includes using distributive practice that is meaningful and purposeful.

Problem-based instructional tasks are at the heart of teaching for understanding. Educators should select high-quality materials around rich instructional tasks focusing on essential mathematics.

Problem-based instructional tasks:

- Help students develop a deep understanding of essential mathematics.
- Emphasize connections across mathematical content areas, to other disciplines, and especially to the real world.
- Are accessible yet challenging to all. Are accessible yet challenging to all.
- Are tasks that students can solve in several ways.
- Encourage student engagement and communication.
- Encourage the use of connected multiple representations.
- Encourage appropriate use of intellectual, physical, and technological tools.

Distributed Practice that is meaningful and purposeful. Practice is essential to learn mathematics. However, to be effective in improving student achievement, practice must be meaningful, purposeful, and distributed.

- Meaningful: Builds on and extends understanding.
- Purposeful: Links to curriculum goals and targets an identified need based on multiple data sources.
- Distributed: Consists of short periods of systematic practice spread over a long period.

(adapted from Iowa Mathematics Standards, 2010)

Standards for Mathematical Practice: Grades K–5

SMP 1: Make sense of problems and persevere in solving them.

Mathematically proficient elementary students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. For example, instead of hunting for “key words” in a word problem, young students might use concrete objects or pictures to show the actions of a problem, such as counting out and joining two sets. If students are not at first making sense of a problem or seeing a way to begin, they ask questions to help get started. As they work, they continually ask themselves, “Does this make sense?” When they find that their solution pathway does not make sense, they look for another pathway that does. They may consider simpler forms of the original problem; for example, they might replace multi-digit numbers in a word problem with single-digit numbers to better appreciate the quantities in the problem and how they relate. Once students have a solution, they often check their answers to problems using a different approach. Mathematically proficient students consider different solution pathways, both their own and those of other students, to identify and analyze correspondences among approaches. They can explain correspondences among physical models, pictures, diagrams, equations, verbal descriptions, tables and graphs.

MP 2: Reason abstractly and quantitatively

Mathematically proficient elementary students make sense of quantities and their relationships in problem situations. They can contextualize quantities and operations by using images or stories. They interpret symbols as having meaning, not just as directions to carry out a procedure. Even as they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects. They can contextualize an abstract problem by placing it in a context they then use to make sense of the mathematical ideas. For example, if a student chooses to evaluate the expression $40 - 26$ mentally, the student might think of a context to help produce a strategy - for example, by thinking “if I have 26 marbles and Marie has 40, how many more do I need to have as many as Marie?” This prompts a strategy of thinking “Four more will get me to a total of 30, and then 10 more will get me to 40, so the answer is 14.” In this example, the student uses a context to think through a strategy for solving the problem, using the relationship between addition and subtraction and decomposing and recomposing the quantities. The student then uses what they did in the context to identify the solution of the original abstract problem. Mathematically proficient students can also make sense of a contextual problem and express the actions or events that are described in the problem using numbers and symbols. If they work with the symbols to solve the problem, they can then interpret their solution in terms of the context.

SMP 3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient elementary students construct verbal and written mathematical arguments that explain the reasoning underlying a strategy, solution or conjecture. Arguments might use concrete referents such as objects, drawings, diagrams, and actions. Arguments might also rely on definitions, previously established results, properties, or structures. For example, a student might argue that $\frac{1}{41} > \frac{1}{73}$ on the basis that one of 41 equal parts of a whole is larger than one of 73 equal parts of that whole; or that two different shapes have equal area because it has already been shown that both shapes are half of the same rectangle. Students might also use counterexamples to argue that a conjecture is not true—for example, a rhombus is an example that shows that not all quadrilaterals with four equal sides are squares; or, multiplying by one shows that a product of two whole numbers is not always greater than each factor. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). In the elementary grades, arguments are often a combination of all three. Some of their arguments apply to individual problems, but others are about conjectures based on regularities they have noticed across multiple problems (see SMP 8). As they articulate and justify generalizations, students consider to which mathematical objects (numbers or shapes, for example) their generalizations apply. For example, young students may believe a generalization about the behavior of addition applies to positive whole numbers less than 100 because those are the numbers with which they are currently familiar. As they expand their understanding of the number system, they reexamine their conjecture for numbers in the hundreds and thousands. In upper elementary grades, students return to their conjectures and arguments about whole numbers to determine whether they apply to fractions and decimals. Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. They can communicate their arguments both orally and in writing, compare them to others, and reconsider their own arguments in response to the critiques of others.

SMP 4: Model with mathematics.

When given a problem in a contextual situation, mathematically proficient elementary students can identify the mathematical elements of a situation and create or interpret a mathematical model that shows those elements and relationships among them. The mathematical model might be represented in one or more of the following ways: numbers and symbols; geometric figures, pictures or physical objects used to abstract the mathematical elements of the situation; a mathematical diagram such as a number line, table, or graph; or students might use more than one of these to help them interpret the situation. For example, when students encounter situations, such as sharing a pan of cornbread among six people, they might first show how to divide the cornbread into six equal pieces using a picture of a rectangle. The rectangle divided into six equal pieces is a model of the essential mathematical elements of the situation. When the students learn to write the name of each piece in relation to the whole part as $\frac{1}{6}$, they are now modeling the situation with mathematical notation. Mathematically proficient students are able to identify important quantities in a contextual situation and use mathematical models to show the relationships of those quantities, particularly in multi-step problems such as those involving more than one unknown quantity. Mathematically proficient students use and interpret models to analyze relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (SMP 2).

Note: Although physical objects and drawings can be used to model a situation, using these tools absent a contextual situation is not an example of SMP 4. For example, using manipulatives or drawing tens and ones to illustrate the calculation $23 + 11$ would not be an example of this mathematical practice. SMP 4 is about applying math to a problem in context.

SMP 5: Use appropriate tools strategically.

Mathematically proficient elementary students consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. These tools might include physical objects (cubes, geometric shapes, place value manipulatives, fraction bars; etc.); drawings or diagrams (number lines, tally marks, tape diagrams, arrays, tables, graphs, etc.); concrete objects that represent mathematical concepts, paper and pencil, rulers and other measuring tools, scissors, tracing paper, grid paper, tactile and virtual manipulatives, appropriate software applications, or other available technologies. Example: a student may use graph paper to find all the possible rectangles that have a given perimeter or use linking cubes to represent two quantities and then compare the two representations side by side. Proficient students are sufficiently familiar with tools appropriate for their grade and areas of content to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from their use as well as their limitations. Students choose tools that are relevant and useful to the problem at hand. Once efficient and generalizable methods are available (such as applying properties of operations and concepts of place value or using the standard written algorithms in the grades indicated in the content standards), being strategic implies that students no longer choose diagrams or concrete objects as tools for calculation. Such representations remain useful as ways to make mathematical thinking visible (SMP 3).

SMP 6: Attend to precision.

Mathematically proficient elementary students communicate precisely to others both verbally and in writing. They start by using everyday language to express their mathematical ideas, realizing that they need to select words with clarity and specificity rather than saying, for example, “it works” without explaining what "it” means. As they encounter the ambiguity of everyday terms, they come to appreciate, understand, and use mathematical vocabulary. Once young students become familiar with a mathematical idea or object, they are ready to learn more precise mathematical terms to describe it. In using mathematical representations, students use care in providing proper labels to precisely communicate the meaning of their representations. When making mathematical arguments about a solution, strategy, or conjecture (see SMP 3), mathematically proficient students learn to craft careful explanations that communicate their reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to their representations. Elementary students use mathematical symbols correctly and can describe the meaning of the symbols they use. When measuring, mathematically proficient students use tools and strategies to minimize the introduction of error. Mathematically proficient students specify units of measure, label charts, graphs, and drawings; calculate accurately and efficiently; and use clear and concise notation to record their work. Diligence and attention to detail are mathematical virtues; mathematically proficient students care that an answer is right; they check their work; they solve the problem another way; they demonstrate accountability by identifying, correcting, and learning from their mistakes.

SMP 7: Look for and make use of structure.

Mathematically proficient elementary students use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, even numbers can be divided into 2 equal groups and odd numbers, when divided by two, always have one left over), and attributes of shapes to solve problems. In many cases, they have identified and described these structures through repeated reasoning (see SMP 8). For example, when younger students recognize that adding one results in the next counting number, they identify the basic structure of whole numbers. When older students calculate 16×9, they might apply the structure of place value and the distributive property to find the product: $16 \times 9 = (10 + 6) \times 9 = (10 \times 9) + (6 \times 9)$. To determine the volume of a $3 \times 4 \times 5$ rectangular prism, students might see the structure of the prism as five layers of 3×4 arrays of cubes. Students in elementary grades most often look for and make use of structure when they view expressions as objects to observe and interpret, for example by observing that $120 - 41$ must be one less than $120 - 40$ because “if students subtract one more, the result will be one less”; or by making sense of $(10 \times 9) + (6 \times 9)$ as “ten nines and six more nines” instead of only being able to see (10×9) as instructions to calculate these products’ values. A word problem that involves distributing 29 marbles among four vases could lead (SMP 4) to an equation model $(29 - 1) \div 4 = 7$, where the expression on the left-hand side not only has the value “7” but also suggests, based on its structure, a process of discarding one marble and dividing the rest of the marbles equally into four groups of seven.

SMP 8: Look for and express regularity in repeated reasoning.

Mathematically proficient elementary students look for regularities as they solve multiple related problems, then identify and describe these regularities. For example, students might notice a pattern in the change to the product when a factor is increased by one: $5 \times 7 = 35$ and $5 \times 8 = 40$—the product changes by five; $9 \times 4 = 36$ and $10 \times 4 = 40$—the product changes by four. Students might then express this regularity by saying something like, “When students change one factor by one, the product increases by the other factor.” Younger students might notice that when tossing two-color counters to find combinations of a given number, they always get what they call “opposites”—when tossing six counters, they get two red, four yellow and four red two yellow and when tossing four counters, they get one red, three yellow and three red, one yellow. Mathematically proficient students formulate conjectures about what they notice, for example, when one is added to a factor, the product increases by the other factor. As students practice articulating their observations both verbally and in writing, they learn to communicate with greater precision (SMP 6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (SMP 3).

(adapted from Iowa Mathematics Standards, 2010)

Kindergarten

Narrative

In kindergarten, instructional time should focus on two major areas:

1. Foundation of numbers including the principle elements of counting and place value.

   The kindergarten math standards focus on developing a solid foundation in early mathematical concepts. In kindergarten, students are expected to develop a deep understanding of numbers and counting. They learn to count to 100 by ones and tens, recognize and write numbers 020, and understand the concept of one-to-one correspondence and one more, one less. Basic addition and subtraction concepts are introduced, such as combining and separating sets of objects. Students also begin to explore shapes, patterns, and measurements, including comparing the length, weight, and capacity of objects. Additionally, they engage in problem-solving activities that encourage critical thinking and mathematical reasoning.

2. Describing the physical world with mathematical vocabulary.

   Kindergarten math standards also emphasize developing strong mathematical communication skills. Students are encouraged to express their mathematical thinking orally and in writing and use math vocabulary to describe and explain their reasoning. They work collaboratively to solve problems and engage in hands-on activities to reinforce their understanding of mathematical concepts. Overall, the kindergarten math standards provide a solid foundation for future mathematical learning and promote mathematical literacy from an early age.

Grade K Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Counting and Cardinality (CC)

- **Know number names and the count sequence forward.**
- Know number names and the count sequence backward.
- **Count to tell the number of objects.**
- **Compare numbers.**

Operations and Algebraic Thinking (OA)

- **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**

Number and Operations in Base Ten (NBT)

- **Work with numbers 11–19 to gain foundations for place value.**

Measurement and Data (MD)

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in each category.
- Identify attributes and values of money.

Geometry (G)

- Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
- Analyze, compare, create and compose shapes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

End of Grade Level Fluency Table

Standard	Fluencies
K.OA.A.5	Add/subtract within 5
1.OA.C.6	Add/subtract within 10
2.OA.B.2	Add and subtract within 20
2.NBT.B.5	Add/subtract within 100
3.OA.C.7	Multiply/divide within 100
3.NBT.A.2	Add/subtract within 1,000
4.NBT.B.4	Add/subtract within 1,000,000
5.NBT.B.5	Multi-digit multiplication
6.NS.B.2	Multi-digit division

Domain: K.CC Counting and Cardinality

K.CC.A Major Cluster: Know number names and the count sequence forward.

Standard	Rigor	SMP Bundle
K.CC.A.1 Count to 100 by ones and by tens.	Procedural	Communicating Reasoning Modeling & Data Analysis
K.CC.A.2 Count forward beginning from any given number within the range of 0–100.	Procedural	Communicating Reasoning
K.CC.A.3 Write numbers from 0 to 20. Given a set of 0–20 objects, write a numeral to represent the quantity.	Conceptual Procedural	Communicating Reasoning

K.IA.A Supporting Cluster: Know number names and the count sequence backward.

Standard	Rigor	SMP Bundle
K.CC.IA.A.1 Count backwards by ones from 20 to 0.	Procedural	Communicating Reasoning
K.CC.IA.A.2 Count backwards beginning from any given number within the range of 0–20.	Procedural	Communicating Reasoning

K.CC.B Major Cluster: Count to tell the number of objects.

Standard	Rigor	SMP Bundle
K.CC.B.4 Demonstrate awareness of the principles of counting. a. Number names must be said in the standard order (sequencing). b. Each object must be paired with one and only one number name and each number name with one and only one object (one-to-one correspondence). c. The last number name said tells the number of objects counted, objects may be counted in any order (cardinality). d. The number of objects is the same regardless of their arrangement or the order in which they were counted (conservation of number). e. Each successive number name refers to a quantity that is one larger.	Conceptual Procedural	Communicating Reasoning
K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration given. Given a number from 1–20, count out that many objects.	Conceptual Procedural	Communicating Reasoning
K.CC.IA.B.1 Quickly recognize and name the quantity of up to 5 objects briefly shown in structured or unstructured arrangements without counting (perceptual subitizing).	Conceptual	Communicating Reasoning

K.CC.C Major Cluster: Compare numbers.

Standard	Rigor	SMP Bundle
K.CC.C.6 Determine whether the number of objects in one group of 1–10 objects is greater than, less than, or equal to the number of objects in another group of 1–10 objects. *For example, using matching and counting strategies.*	Conceptual	Communicating Reasoning
K.CC.C.7 Compare two numbers between 1 and 10 presented as written numerals.	Conceptual Procedural	Communicating Reasoning

Domain: K.OA Operations and Algebraic Thinking:

K.OA.A Major Cluster: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Standard	Rigor	SMP Bundle
K.OA.A.1 Represent addition and subtraction situations in a variety of ways. *For example, with objects, fingers, mental images, drawings, sounds (claps), acting out situations, verbal explanations, expressions, or equations.*	Conceptual	Communicating Reasoning
K.OA.A.2 Add and subtract within 10 and solve word problems involving the different problem types listed below. *For example, by using objects or drawings to represent the problem.* • Add-to with result unknown. • Take-from with result unknown. • Put-together/take-apart with total unknown. • Put-together with both addends unknown. **(See Problem Types Table 1 in Appendix A)**	Application	Problem Solving Modeling & Data Analysis
K.OA.A.3 Decompose numbers less than or equal to 10 in more than one way. *For example, by using objects or drawings, and record each decomposition by a drawing or equation, as in* $5 = 2 + 3$, $5 = 4 + 1$, and $5 = 2 + 2 + 1$.	Conceptual Procedural	Communicating Reasoning
K.OA.A.4 For any number from 1 to 9, find the number that makes 10 when added to the given numbers by using objects or drawings and record the answer with a drawing or equation.	Conceptual Procedural	Communicating Reasoning
K.OA.A.5 Fluently add and subtract within 5 using efficient mental strategies. • Counting on. • Counting back. • Using the relationship between addition and subtraction. • Creating equivalent, but easier or known sums. By the end of kindergarten, flexibly, efficiently and accurately find all sums within 5. Note: Fluency of this standard is critical by the end of grade level.	Conceptual Procedural	Communicating Reasoning

Domain: K.NBT Number and Operations in Base Ten

K.NBT.A Major Cluster: Work with numbers 11–19 to gain foundations for place value.

Standard	Rigor	SMP Bundle
K.NBT.A.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, by using objects or drawings, and record each composition or decomposition by a drawing or equation. For example, $18 = 10 + 8$; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	Conceptual	Communicating Reasoning

Domain: K.MD Measurement and Data

K.MD.A Additional Cluster: Describe and compare measurable attributes.

Standard	Rigor	SMP Bundle
K.MD.A.1 Describe several measurable attributes (*for example, length, width, weight*) of objects by using words such as short, long, small, big, heavy, light.	Conceptual	Communicating Reasoning
K.MD.A.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	Conceptual Application	Communicating Reasoning Modeling & Data Analysis

K.MD.B Supporting Cluster: Classify objects and count the number of objects in each category.

Standard	Rigor	SMP Bundle
K.MD.B.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. Limit category counts to be less than or equal to 10.	Conceptual Application	Communicating Reasoning

K.MD.IA.B Additional Cluster: Identify attributes and values of money.

Standard	Rigor	SMP Bundle
K.MD.IA.B.1 Identify the penny and know the value is one cent. Count pennies up to 20.	Procedural Conceptual Application	Communicating Reasoning

Domain: K.G Geometry

K.G.A Additional Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

Standard	Rigor	SMP Bundle
K.G.A.1 Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, besides, in front of, behind, and next to.	Conceptual Procedural Application	Communicating Reasoning
K.G.A.2 Correctly name shapes regardless of their orientations or overall size.	Procedural Application	Communicating Reasoning
K.G.A.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	Procedural Application	Communicating Reasoning

K.G.B Supporting Cluster: Analyze, compare, create, and compose shapes.

Standard	Rigor	SMP Bundle
K.G.B.4 Analyze and compare two- and three-dimensional shapes, in varied sizes and orientations, using informal language to describe their similarities, differences, parts and other attributes. *For example, number of sides and vertices/corners and having sides of equal length*.	Conceptual Application	Communicating Reasoning Modeling & Data Analysis
K.G.B.5 Model shapes in the world by building shapes from components and drawing shapes. For *example, sticks and clay balls*.	Conceptual Application	Communicating Reasoning Modeling & Data Analysis
K.G.B.6 Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	Conceptual Application	Communicating Reasoning Modeling & Data Analysis

First Grade

Narrative

In Grade 1, instructional time should focus on three major areas:

1. Developing an understanding of addition, subtraction, and strategies for addition and subtraction within 20.

   Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (for example, cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand the connections between counting and addition and subtraction (for example, adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (for example, "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. Developing an understanding of whole number relationships and place value, including grouping in tens and ones.

   Students develop, discuss, and use efficient, accurate, and flexible methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to100) to develop an understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Developing an understanding of linear measurement and measuring lengths as iterating length units.

   Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement, which means that if A is longer than B, and B is longer than C, then A is longer than C.

Grade 1 Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Operations and Algebraic Thinking (OA)

- **Represent and solve problems involving addition and subtraction.**
- **Understand and apply properties of operations and the relationship between addition and subtraction.**
- **Add and subtract within 20.**
- **Work with addition and subtraction equations.**

Number and Operations in Base Ten (NBT)

- **Extend the counting sequence.**
- **Understand place value.**
- **Use place value understanding and properties of operations to add and subtract.**

Measurement and Data (MD)

- **Measure lengths indirectly and by iterating length units.**
- Work with time and money.
- Represent and interpret data.

Geometry (G)

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

End of Grade Level Fluency Table

Standard	Fluencies
K.OA.A.5	Add/subtract within 5
1.OA.C.6	Add/subtract within 10
2.OA.B.2	Add and subtract within 20
2.NBT.B.5	Add/subtract within 100
3.OA.C.7	Multiply/divide within 100
3.NBT.A.2	Add/subtract within 1,000
4.NBT.B.4	Add/subtract within 1,000,000
5.NBT.B.5	Multi-digit multiplication
6.NS.B.2	Multi-digit division

Domain: 1.OA Operations and Algebraic Thinking

1.OA.A Major Cluster: Represent and solve problems involving addition and subtraction.

Standard	Rigor	SMP Bundle
1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving the problem types listed below, with unknowns in all positions, by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. • Adding to. • Taking from. • Putting together. • Taking apart. • Comparing. **(See Problem Types Table 1 in Appendix A)**	Conceptual Application	Problem Solving Modeling & Data Analysis
1.OA.A.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20. *For example, by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.*	Conceptual Application	Problem Solving Modeling & Data Analysis

1.OA.B Major Cluster: Understand and apply properties of operations and the relationship between addition and subtraction.

Standard	Rigor	SMP Bundle
1.OA.B.3 Apply properties of operations, (commutative and associative), as strategies to add and subtract. *For example, Commutative property of addition, if* $8+3=11$ *is known then,* $3+8=11$ *is also known. Associative property of addition, to add,* $2+6+4$*, the second two numbers can be added to make a ten, so* $2+6+4=2+10=12$.	Conceptual	Problem Solving
1.OA.B.4 Understand subtraction as an unknown-addend problem. *For example, subtract* $10-8$ *by finding the number that makes 10 when added to 8.*	Conceptual	Communicating Reasoning

1.OA.C Major Cluster: Add and subtract within 20.

Standard	Rigor	SMP Bundle
1.OA.IA.C.1 Use counting and subitizing strategies to explain addition and subtraction. a. Relate counting to addition and subtraction *(for example, by counting on 2 to add 2).* b. Use conceptual subitizing in unstructured arrangements with totals up to 10 and structured arrangements anchored to 5 or 10 (*for example, 10 frames, double ten frames, math rack*) with totals up to 20 to relate the compositions and decompositions to addition and subtraction.	Conceptual	Communicating Reasoning
1.OA.C.5 Relate counting forward and backward to addition and subtraction, add or subtract 1 or 2.	Conceptual	Modeling & Data Analysis
1.OA.C.6 Add and subtract within 20, using strategies such as: • Counting on. • Making ten *(for example,* $8+6=8+2+4=10+4=14$*).* • Decomposing a number leading to a ten *(for example,* $13-4=13-3-1=10-1=9$*).* • Using the relationship between addition and subtraction *(for example, knowing that* $8+4=12$*, one knows* $12-8=4$*).* • Creating equivalent but easier or known sums *(for example, adding* $6+7$ *by creating the known equivalent* $6+6+1=12+1=13$*).* • Counting up to subtract.	Conceptual	Problem Solving
1.OA.IA.C.2 Fluently add and subtract within 10 using efficient mental strategies. • Counting on. • Making ten. • Decomposing a number leading to a ten. • Using the relationship between addition and subtraction. • Creating equivalent, but easier or known sums. • Counting up to subtract. By the end of Grade 1, flexibly, efficiently, and accurately find all sums within 10. Note: Fluency of this standard is critical by the end of grade level.	Conceptual	Problem Solving

1.OA.D Major Cluster: Work with addition and subtraction equations.

Standard	Rigor	SMP Bundle
1.OA.D.7 Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations is true and which is false?* $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 3 = 5 + 2$.	Conceptual Procedural	Communicating Reasoning
1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 + \square = 11$, $5 = \square - 3$, $6 + 6 = \square$.	Procedural	Problem Solving

Domain: 1.NBT Number and Operations in Base Ten

1.NBT.A Major Cluster: Extend the counting sequence.

Standard	Rigor	SMP Bundle
1.NBT.A.1 Count forward and backward starting with any given number within the range of 0–120, In this range, read and write numerals and represent a number of objects with a written numeral.	Conceptual Procedural	Modeling & Data Analysis

1.NBT.B Major Cluster: Understand place value.

Standard	Rigor	SMP Bundle
1.NBT.B.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases. a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	Conceptual	Communicating Reasoning
1.NBT.B.3 Compare two two-digit numbers based on meanings of the tens and ones digits, using phrases as greater than, less than or equal to, connecting to the use of >, =, and < symbols.	Conceptual	Communicating Reasoning

1.NBT.C Major Cluster: Use place value understanding and properties of operations to add and subtract.

Standard	Rigor	SMP Bundle
1.NBT.C.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	Conceptual Procedural	Modeling & Data Analysis
1.NBT.C.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	Conceptual Procedural	Communicating Reasoning
1.NBT.C.6 Subtract multiples of 10 in the range 10 to 90 from multiples of 10 in the range 10 to 90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; explain the reasoning used.	Conceptual Procedural	Modeling & Data Analysis

Domain: 1.MD Measurement and Data

1.MD.A Major Cluster: Measure lengths indirectly and by iterating length units.

Standard	Rigor	SMP Bundle
1.MD.A.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.	Conceptual Procedural	Communicating Reasoning
1.MD.A.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	Conceptual Procedural	Communicating Reasoning

1.MD.B Additional Cluster: Work with time and money.

Standard	Rigor	SMP Bundle
1.MD.B.3 Tell and write time in hours and half-hours using analog and digital clocks.	Conceptual Procedural	Modeling & Data Analysis
1.MD.IA.B.1 Identify pennies and dimes and their values. Count a mixed collection of dimes and pennies to determine the cent value (total not to exceed 100 cents).	Conceptual Procedural	Communicating Reasoning Modeling & Data Analysis

1.MD.C Supporting Cluster: Represent and interpret data.

Standard	Rigor	SMP Bundle
1.MD.C.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	Conceptual Procedural	Communicating Reasoning Modeling & Data Analysis

Domain: 1.G Geometry

1.G.A Additional Cluster: Reason with shapes and their attributes.

Standard	Rigor	SMP Bundle
1.G.A.1 Distinguish between defining attributes *(for example, triangles are closed and three-sided)* versus non-defining attributes *(for example, color, orientation, overall size)*; build and draw shapes to possess defining attributes.	Conceptual Procedural	Communicating Reasoning
1.G.A.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, rectangular prisms, cones and cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names for these shapes.	Conceptual	Modeling & Data Analysis
1.G.A.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	Conceptual Procedural	Communicating Reasoning Modeling & Data Analysis

Second Grade

In grade 2, instructional time should focus on three major areas:

1. Extending understanding of base-ten system.

   Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1,000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (for example, 853 is 8 hundreds + 5 tens + 3 ones).

2. Building fluency with addition and subtraction.

   Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1,000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and flexible methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply appropriate methods for the context and the numbers involved to accurately calculate sums and differences for numbers with only tens or only hundreds.

3. Using standard units of measure.

   Students recognize the need for standard units of measure (centimeter and inch), and they use rulers and other measurement tools with the understanding that linear measurement involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

Grade 2 Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Operations and Algebraic Thinking (OA)

- **Represent and solve problems involving addition and subtraction.**
- **Add and subtract within 20.**
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten (NBT)

- **Understand place value.**
- **Use place value understanding and properties of operations to add and subtract.**

Measurement and Data (MD)

- **Measure and estimate lengths in standard units.**
- **Relate addition and subtraction to length.**
- Work with time and money.
- Represent and interpret data.

Geometry (G)

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

End of Grade Level Fluency Table

Standard	Fluencies
K.OA.A.5	Add/subtract within 5
1.OA.C.6	Add/subtract within 10
2.OA.B.2	Add and subtract within 20
2.NBT.B.5	Add/subtract within 100
3.OA.C.7	Multiply/divide within 100
3.NBT.A.2	Add/subtract within 1,000
4.NBT.B.4	Add/subtract within 1,000,000
5.NBT.B.5	Multi-digit multiplication
6.NS.B.2	Multi-digit division

Domain: 2.OA Operations and Algebraic Thinking

2.OA.A Major Cluster: Represent and solve problems involving addition and subtraction.

Standard	Rigor	SMP Bundle
2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving the problem types listed below, with unknowns in all positions, by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. • Adding to. • Taking from. • Putting together. • Taking apart. • Comparing. **(See Problem Types Table 1 in Appendix A)**	Conceptual Application	Problem Solving Modeling & Data Analysis

2.OA.B Major Cluster: Add and Subtract within 20.

Standard	Rigor	SMP Bundle
2.OA.B.2 Fluently add and subtract within 20 using efficient mental strategies listed below. • Counting on. • Counting back. • Making ten. • Decomposing a number leading to a ten. • Using the relationship between addition and subtraction. • Creating equivalent, but easier or known sums. • Adding up to subtract. By the end of Grade 2, flexibly, efficiently and accurately find all sums of two one-digit numbers. Note: Fluency of this standard is critical by the end of grade.	Procedural	Problem Solving Modeling & Data Analysis

2.OA.C Supporting Cluster: Work with equal groups of objects to gain foundations for multiplication.

Standard	Rigor	SMP Bundle
2.OA.C.3 Determine whether a group of objects (up to 20) has an odd or even number of members; write an equation to express an even number as a sum of two equal addends. *For example, by pairing objects or counting them by 2s.*	Conceptual	Modeling & Data Analysis
2.OA.C.4 Use repeated addition to find the total number of objects arranged in equal groups and rectangular arrays; write an equation to express the total as a sum of equal addends.	Conceptual	Modeling & Data Analysis

Domain: 2.NBT Number and Operations in Base Ten

2.NBT.A Major Cluster: Understand place value.

Standard	Rigor	SMP Bundle
2.NBT.A.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. Understand the following as special cases: • 100 can be thought of as a bundle of ten tens — called a "hundred." • The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and zero tens and zero ones). *706 equals 7 hundreds, 0 tens, and 6 ones.*	Conceptual	Communicating Reasoning Modeling & Data Analysis
2.NBT.A.2 Count forward and backward within 1,000; skip-count forward and backward by 5s, 10s, and 100s.	Procedural	Modeling & Data Analysis
2.NBT.A.3 Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form.	Conceptual	Modeling & Data Analysis
2.NBT.A.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using terms “greater than”, “less than”, and “equal to”, connecting to the use of >, =, and < symbols.	Conceptual	Modeling & Data Analysis

2.NBT.B Major Cluster: Use place value understanding and properties of operations to add and subtract.

Standard	Rigor	SMP Bundle
2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Note: Fluency of this standard is critical by the end of grade level.	Conceptual Procedural	Problem Solving Modeling & Data Analysis
2.NBT.B.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.	Conceptual Procedural	Problem Solving Modeling & Data Analysis
2.NBT.B.7 Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	Conceptual Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis
2.NBT.B.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.	Procedural	Problem Solving
2.NBT.B.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.	Conceptual	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: 2.MD Measurement and Data

2.MD.A Major Cluster: Measure and estimate lengths in standard units.

Standard	Rigor	SMP Bundle
2.MD.A.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	Conceptual Procedural	Modeling & Data Analysis
2.MD.A.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	Conceptual Procedural	Communicating Reasoning Modeling & Data Analysis
2.MD.A.3 Estimate lengths using units of inches, feet, centimeters, and meters.	Conceptual	Communicating Reasoning Modeling & Data Analysis
2.MD.A.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard-length unit.	Conceptual Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

2.MD.B Major Cluster: Relate addition and subtraction to length.

Standard	Rigor	SMP Bundle
2.MD.B.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units. *For example, by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.*	Procedural Application	Problem Solving Modeling & Data Analysis
2.MD.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.	Conceptual Application	Modeling & Data Analysis

2.MD.C Supporting Cluster: Work with time and money.

Standard	Rigor	SMP Bundle
2.MD.C.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	Procedural Application	Communicating Reasoning Modeling & Data Analysis
2.MD.IA.C.1 Describe the relationship among standard units of time: minutes, hours, days, weeks, months and years (such as 7 days in a week, 60 minutes in an hour, etc.).	Conceptual Procedural	Communicating Reasoning Modeling & Data Analysis
2.MD.IA.C.2 Identify nickels, quarters and dollars and know their values.	Conceptual Procedural	Communicating Reasoning Modeling & Data Analysis
2.MD.C.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *For example, if you have 3 quarters, 2 dimes and 4 pennies, how many cents do you have?* For this standard, it may be appropriate to record amounts using decimals but does not include adding and subtracting with decimals.	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

2.MD.D Supporting Cluster: Represent and interpret data.

Standard	Rigor	SMP Bundle
2.MD.D.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	Procedural Application	Communicating Reasoning Modeling & Data Analysis
2.MD.IA.D.1 Use interviews, surveys, and observations to collect data that answer questions about students' interests and/or their environment.	Application	Modeling & Data Analysis
2.MD.D.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple problems: put-together, take-apart, and compare, using information presented in a bar graph. **(See Problem Types Table 1 in Appendix A)**	Procedural Application	Problem Solving Modeling & Data Analysis

Domain: 2.G Geometry

2.G.A Additional Cluster: Reason with shapes and their attributes.

Standard	Rigor	SMP Bundle
2.G.A.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify two-dimensional shapes: triangles, quadrilaterals, rectangles, squares, trapezoids, pentagons, hexagons, circles, half-circles and quarter-circles, and three-dimensional figures: cubes, right rectangular prisms, right circular cones, and right circular cylinders. (Sizes are compared directly or visually, not compared by measuring.)	Conceptual	Modeling & Data Analysis
2.G.A.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of squares.	Conceptual	Modeling & Data Analysis
2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	Conceptual	Modeling & Data Analysis

Third Grade

In Grade 3, instructional time should focus on three major areas:

1. Gaining an understanding of multiplication and division and strategies for multiplication and division within 100.

   Students gain an understanding of multiplication and division and strategies for multiplication and division within 100. This involves learning the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. Gaining an understanding of fractions, especially unit fractions (fractions with numerator 1).

   Students gain an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, $\frac{1}{2}$ of the paint in a small bucket could be less paint than $\frac{1}{3}$ of the paint in a larger bucket, but $\frac{1}{3}$ of a ribbon is longer than $\frac{1}{5}$ of the same ribbon because when the ribbon is divided into three equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students can use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Gaining an understanding of the structure of rectangular arrays and area.

   Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area needed to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication and justify using multiplication to determine the area of a rectangle.

Grade 3 Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Operations and Algebraic Thinking (OA)

- **Represent and solve problems involving multiplication and division.**
- **Use properties of operations and the relationship between multiplication and division.**
- **Multiply and divide within 100.**
- **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**

Number and Operations in Base Ten (NBT)

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions (NF)

- **Understand fractions as numbers.**

Measurement and Data (MD)

- **Solve problems with time, money and measured quantities.**
- Represent and interpret data.
- **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry (G)

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

End of Grade Level Fluency Table

Standard	Fluencies
K.OA.A.5	Add/subtract within 5
1.OA.C.6	Add/subtract within 10
2.OA.B.2	Add and subtract within 20
2.NBT.B.5	Add/subtract within 100
3.OA.C.7	Multiply/divide within 100
3.NBT.A.2	Add/subtract within 1,000
4.NBT.B.4	Add/subtract within 1,000,000
5.NBT.B.5	Multi-digit multiplication
6.NS.B.2	Multi-digit division

Domain: 3.OA Operations and Algebraic Thinking

3.OA.A Major Cluster: Represent and solve problems involving multiplication and division.

Standard	Rigor	SMP Bundle
3.OA.A.1 Interpret products of whole numbers. *For example, interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each; describe a context in which a total number of objects can be expressed as 5 × 7.*	Conceptual	Problem Solving Modeling & Data Analysis
3.OA.A.2 Interpret whole-number quotients of whole numbers as the number of groups or the number in each group in situations of equal groups. *For example, describe a context involving equal groups of objects in which the number of groups or the number in each group can be expressed as 56 ÷ 8*. **(See Problem Types Table 2 in Appendix A)**	Conceptual	Problem Solving Modeling & Data Analysis
3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, with unknowns in all positions. *For example, by using drawings and equations with a symbol for the unknown number to represent the problem.* **(See Problem Types Table 2 in Appendix A)**	Conceptual Application	Problem Solving Modeling & Data Analysis
3.OA.A.4 Be able to represent a word problem by writing an equation with a symbol for the unknown whole number and determine the unknown whole number in a multiplication or division equation relating to three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times \square = 48$, $5 = \square \div 3$, $6 \times 6 = \square$.	Conceptual Procedural	Problem Solving Modeling & Data Analysis

3.OA.B Major Cluster: Use properties of operations and the relationship between multiplication and division.

Standard	Rigor	SMP Bundle
3.OA.B.5 Use properties of operations as strategies to multiply and divide. *For example, if* $6 \times 4 = 24$ *is known, then* $4 \times 6 = 24$ *is also known. (Commutative property of multiplication.)* $3 \times 5 \times 2$ *can be found by* $3 \times 5 = 15$, *then* $15 \times 2 = 30$, *or by* $5 \times 2 = 10$, *then* $3 \times 10 = 30$. *(Associative property of multiplication.) Knowing that* $8 \times 5 = 40$ *and* $8 \times 2 = 16$, *one can find* 8×7 *as* $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. *(Distributive property.)*	Conceptual	Communicating Reasoning
3.OA.B.6 Understand division as an unknown-factor problem. *For example, find* $32 \div 8$ *by finding (or remembering) the number that makes 32 when multiplied by* 8 ($\square \times 8 = 32$).	Conceptual	Communicating Reasoning

3.OA.C Major Cluster: Multiply and divide within 100.

Standard	Rigor	SMP Bundle
3.OA.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division. *For example, knowing that* $8 \times 5 = 40$*, one knows* $40 \div 5 = 8$*) or properties of operations.* By the end of Grade 3, flexibly, efficiently, and accurately find all products of two one-digit numbers. Note: Fluency of this standard is critical by the end of grade level.	Procedural	Problem Solving

3.OA.D Major Cluster: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Standard	Rigor	SMP Bundle
3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. Note: this standard is limited to problems posed with whole numbers and have whole number answers; students should know how to perform operations in conventional order when there are no parentheses to specify a particular order (Order of Operations).	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis
3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	Conceptual Application	Communicating Reasoning Problem Solving Modeling & Data Analysis

Domain: 3.NBT Number and Operations in Base Ten

3.NBT.A Additional Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic**.**

Standard	Rigor	SMP Bundle
3.NBT.A.1 Round whole numbers to the nearest 10 or 100 within the range of 0–1,000. *For example, rounding 643 to the nearest 10 would be 640; to the nearest 100 would be 600.*	Conceptual	Problem Solving
3.NBT.A.2 Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. *For example, 412 - 13 =412 -12 -1 = 400 -1 =399; 505+70 = 575*. Note: Fluency of this standard is critical by the end of grade level.	Conceptual Procedural	Problem Solving
3.NBT.A.3 Use place value and properties of operations to multiply one-digit whole numbers by multiples of 10 in the range 10–90. *For example,* 9×80, 5×60.	Conceptual Procedural	Problem Solving

Domain: 3.NF Number and Operations - Fractions

Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

3.NF.A Major Cluster: Understand fractions as numbers.

Standard	Rigor	SMP Bundle
3.NF.A.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by *a* part of size $\frac{1}{b}$.	Conceptual	Communicating Reasoning
3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line. b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.	Conceptual	Communicating Reasoning
3.NF.A.3 Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions. *For example,* $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent. *For example, by using a visual fraction model.* c. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *For example, express 3 in the form* $3 = \frac{3}{1}$*; recognize that* $\frac{6}{1} = 6$*; locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusion. *For example, by using a visual fraction model.*	Conceptual	Communicating Reasoning

Domain: 3.MD Measurement and Data

3.MD.A Major Cluster: Solve problems with time, money, and measured quantities.

Standard	Rigor	SMP Bundle
3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes. *For example, by representing the problem on a number line.*	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
3.MD.A.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l) (Excludes compound units such as cubic centimeters and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving measured quantities (masses and liquid volumes). Excludes multiplicative comparison problems involving notions of "times as much"; problems do not require unit conversion. **(See Problem Types Table 2 in Appendix A)**)	Conceptual Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

3.MD.B Supporting Cluster: Represent and Interpret Data.

Standard	Rigor	SMP Bundle
3.MD.B.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.	Conceptual Application	Problem Solving
3.MD.B.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	Procedural Application	Problem Solving

3.MD.C Major Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Standard	Rigor	SMP Bundle
3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	Conceptual	Problem Solving Modeling & Data Analysis
3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	Conceptual Procedural	Problem Solving Modeling & Data Analysis
3.MD.C.7 Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of figures that can be decomposed into non-overlapping rectangles and add the areas of the non-overlapping parts, applying this technique to solve real world problems.	Conceptual	Problem Solving Communicating Reasoning Modeling & Data Analysis

3.MD.D Additional Cluster: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Standard	Rigor	SMP Bundle
3.MD.D.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

Domain: 3.G Geometry

3.G.A Supporting Cluster: Reason with shapes and their attributes.

Standard	Rigor	SMP Bundle
3.G.A.1 Understand that shapes in different categories *(for example, rhombuses, rectangles, and others)* may share attributes *(for example, having four sides)*, and that the shared attributes can define a larger category *(for example, quadrilaterals)*. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	Conceptual	Communicating Reasoning
3.G.A.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as* $\frac{1}{4}$ *of the area of the shape.*	Conceptual Procedural	Communicating Reasoning

Fourth Grade

Narrative

In Grade 4, instructional time should focus on two major areas:

1. Developing understanding and fluency with multi-digit multiplication and developing an understanding of dividing to find quotients involving multi-digit dividends.

   Students generalize their understanding of place value to 1,000,000, seeing that each place value is ten times the value of the place value to the right. Students extend their understanding of addition and subtraction using the standard algorithm to find larger sums and differences. They apply their understanding of models, place value, and the distributive property as they discuss and use efficient, accurate, and generalizable methods to fluently compute products and quotients of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate and mentally calculate products and quotients. Students interpret remainders based on the context.

2. Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.

   Students develop an understanding of fraction equivalence and operations with fractions. Students use methods for generating equivalent fractions. Students learn how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

Grade 4 Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Operations and Algebraic Thinking (OA)

- **Use the four operations with whole numbers to solve problems.**
- Gain familiarity with factors and multiples.
- Analyze a number sequence that follows a given rule.

Number and Operations in Base Ten (NBT)

- **Generalize place value understanding for multi-digit whole numbers up to 1,000,000.**
- **Calculate with multi-digit numbers.**

Number and Operations—Fractions (NF)

- **Extend understanding of fraction equivalence and ordering.**
- **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**
- **Understand decimal notation for fractions for tenths and hundredths.**

Measurement and Data (MD)

- Solve problems involving conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data using a line plot.
- Understand the concept of angle and measure angles.

Geometry (G)

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

End of Grade Level Fluency Table

Standard	Fluencies
K.OA.A.5	Add/subtract within 5
1.OA.C.6	Add/subtract within 10
2.OA.B.2	Add and subtract within 20
2.NBT.B.5	Add/subtract within 100
3.OA.C.7	Multiply/divide within 100
3.NBT.A.2	Add/subtract within 1,000
4.NBT.B.4	Add/subtract within 1,000,000
5.NBT.B.5	Multi-digit multiplication
6.NS.B.2	Multi-digit division

Domain: 4.OA Operations and Algebraic Thinking

4.OA.A Major Cluster: Use the four operations with whole numbers to solve problems.

Standard	Rigor	SMP Bundle
4.OA.A.1 Interpret a multiplication equation as a comparison and represent verbal statements of multiplicative comparisons as multiplication equations. *For example, write 35 = 7 x 5 to represent the statement that a 35-foot-long whale shark is 7 times as long as a 5-foot-long reef shark.* **(See Problem Types Table 2 in Appendix A)**	Conceptual	Problem Solving Modeling & Data Analysis
4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison. Be able to use drawings and equations with a variable for the unknown number to represent the problem. *For example, Tom's pencil is 4 times as long as Julie's pencil. Tom's pencil is 8 inches long. How long is Julie's pencil? (multiplicative comparison) For example, Julie's pencil is 2 inches long. Tom's pencil is 8 inches long. How much longer is Tom's pencil than Julie's pencil? (additive comparison)* **(See Problem Types Table 2 in Appendix A)**	Conceptual Application	Problem Solving Modeling & Data Analysis
4.OA.A.3 Solve multistep word problems posed with whole numbers and whole-number answers using the four operations, including problems in which remainders must be interpreted. Be able to represent word problems with math diagrams and with equations in which a letter stands for an unknown quantity and be able to assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Conceptual Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

4.OA.B Supporting Cluster: Gain familiarity with factors and multiples.

Standard	Rigor	SMP Bundle
4.OA.B.4 Be able to find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	Conceptual Procedural	Communicating Reasoning

4.OA.C Additional Cluster: Analyze a number sequence that follows a given rule.

Standard	Rigor	SMP Bundle
4.OA.C.5 Given the rule for a sequence of numbers, identify apparent features of the sequence that were not explicit in the rule itself; explain informally why the numbers will continue to alternate in this way. *For example, given the rule "Add 3" and the number sequence 1, 4, 7, 10, 13 observe that the terms appear to alternate between odd and even numbers;*	Conceptual Procedural	Communicating Reasoning

Domain: 4.NBT Numbers and Operations in Base Ten

4.NBT.A Major Cluster: Generalize place value understanding for multi-digit whole numbers up to 1,000,000.

Standard	Rigor	SMP Bundle
4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what that same digit represents in the place to its right. *For example, recognize that* $700 \div 70 = 10$ *by applying concepts of place value and division.*	Conceptual	Communicating Reasoning
4.NBT.A.2 Read and write whole multi-digit numbers using base-ten numerals (standard form), number names (word form), and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Conceptual Procedural	Communicating Reasoning
4.NBT.A.3 Use place value understanding to round multi-digit whole numbers to any place. *For example, 435,450 rounded to the nearest ten-thousands place is 440,000 because it is more than halfway between 430,000 and 440,000.*	Conceptual Procedural	Communicating Reasoning

4.NBT.B Major Cluster: Calculate with multi-digit numbers.

Standard	Rigor	SMP Bundle
4.NBT.B.4 Fluently add and subtract multi-digit whole numbers up to 1,000,000 using an algorithm. Algorithms may include the standard algorithm, partial sums, partial differences, counting or adding up in increments. Note: Fluency of this standard is critical by the end of grade level.	Procedural	Problem Solving
4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Be able to illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Conceptual Procedural	Problem Solving Communicating Reasoning
4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Conceptual Procedural	Problem Solving Communicating Reasoning

Domain: 4.NF Numbers and Operations–Fractions

Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.A Major Cluster: Extend understanding of fraction equivalence and ordering.

Standard	Rigor	SMP Bundle
4.NF.A.1 Illustrate and explain numerical statements of fraction equivalence by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and write equivalent fractions.	Conceptual	Problem Solving Communicating Reasoning
4.NF.A.2 Compare two fractions with different numerators and different denominators, by creating common denominators or numerators, comparing to a benchmark fraction such as $\frac{1}{2}$ and/or by using a visual fraction model. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.	Conceptual	Problem Solving Communicating Reasoning

4.NF.B Major Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Standard	Rigor	SMP Bundle
4.NF.B.3 Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$. *For example,* $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Be able to justify decompositions. *For example, by using a visual fraction model.* *For example:* $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$. c. Add and subtract mixed numbers with like denominators and show sums and differences of mixed numbers on a number line diagram. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, by using visual fraction models and or equations to represent the problem.	Conceptual Procedural	Problem Solving Modeling & Data Analysis
4.NF.B.4 Apply and extend earlier understandings of multiplication to multiply a fraction by a whole number. a. Using a visual fraction model, understand a fraction with a numerator greater than 1 is a multiple of a unit fraction. *For example, using a number line to show* $\frac{5}{4}$ *as the product of* $5 \times \frac{1}{4}$. b. Multiply a fraction by a whole number using the principle that the product is the whole number times the numerator of the fraction with the same denominator. c. Solve word problems involving multiplication of a fraction by a whole number. Use visual fraction models and/or equations to represent the problem.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

4.NF.C Major Cluster: Understand decimal notation for fractions for tenths and hundredths.

Standard	Rigor	SMP Bundle
4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. *For example, express* $\frac{3}{10}$ *as* $\frac{30}{100}$, *and add* $\frac{3}{10}+\frac{4}{100}=\frac{34}{100}$. Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators is not a requirement at this grade.	Conceptual Procedural	Problem Solving
4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as* $\frac{62}{100}$ *and locate 0.62 on a number line.*	Conceptual Procedural Application	Problem Solving
4.NF.C.7 Compare two decimals to hundredths by reasoning about their size, recording the results of comparisons with the symbols >, =, or <. Recognize that comparisons are valid only when the two decimals refer to the same whole. Show decimals on a number line diagram and be able to justify numerical statements of decimal comparison by using a visual fraction model.	Conceptual	Problem Solving Communicating Reasoning

Domain: 4.MD Measurement and Data

4.MD.A Supporting Cluster: Solve problems involving conversion of measurements from a larger unit to a smaller unit.

Standard	Rigor	SMP Bundle
4.MD.A.1 Know relative sizes of measurement units within one system of measurement, including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit by using multiplication. *For example, record measurement equivalents in a two-column table, know that 1 ft is 12 times as long as 1 in or express the length of a 4 ft snake as 48 in.*	Conceptual Procedural	Problem Solving Modeling & Data Analysis
4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time (including elapsed time), liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.	Conceptual Application	Problem Solving Modeling & Data Analysis
4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	Procedural Application	Problem Solving Modeling & Data Analysis

4.MD.B Supporting Cluster: Represent and interpret data using a line plot.

Standard	Rigor	SMP Bundle
4.MD.B.4 Make a line plot to display a data set of measurements using the fractions of a unit. $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8})$. • $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}$ • $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ • $\frac{1}{2}, \frac{2}{2}$ Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest pencils in a collection.*	Conceptual Procedural Application	Modeling & Data Analysis

4.MD.C Additional Cluster: Geometric measurement: understand the concept of angle and measure angles.

Standard	Rigor	SMP Bundle
4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the angle's rays. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of n°. *For example, an angle that turns through 45 one-degree angles has an angle measure of 45 degrees.*	Conceptual	Problem Solving Modeling & Data Analysis
4.MD.C.6 Draw and measure angles in whole-number degrees (1–180°) using a protractor. Sketch angles of specified measure.	Procedural	Problem Solving Modeling & Data Analysis
4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems. *For example, by using an equation with a symbol for the unknown angle measure.*	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

Domain: 4.G Geometry

4.G.A Additional Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Standard	Rigor	SMP Bundle
4.G.A.1 Draw points, lines, line segments, rays, angles (acute, right, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	Conceptual Procedural	Communicating Reasoning
4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles.	Conceptual Procedural	Communicating Reasoning
4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	Conceptual	Communicating Reasoning

Fifth Grade

In Grade 5, instructional time should focus on two major areas:

1. Develop fluency with addition and subtraction of fractions and developing an understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).

   Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions and make reasonable estimates. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions).

2. Extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.

   Students develop an understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They continue to work with multi-digit addition, subtraction, multiplication, and division. They apply their understanding of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop flexible thinking in these computations and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

Grade 5 Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Operations and Algebraic Thinking (OA)

- Write and interpret numerical expressions.
- Analyze a pair of number sequences.

Number and Operations in Base Ten (NBT)

- **Understand the place value system.**
- **Perform operations with multi-digit whole numbers and with decimals to hundredths.**

Number and Operations—Fractions (NF)

- **Use equivalent fractions as a strategy to add and subtract fractions.**
- **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Measurement and Data (MD)

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- **Geometric measurement: understand concepts of volume and relate volume to multiplication and addition.**

Geometry (G)

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

End of Grade Level Fluency Table

Standard	Fluencies
K.OA.A.5	Add/subtract within 5
1.OA.C.6	Add/subtract within 10
2.OA.B.2	Add and subtract within 20
2.NBT.B.5	Add/subtract within 100
3.OA.C.7	Multiply/divide within 100
3.NBT.A.2	Add/subtract within 1,000
4.NBT.B.4	Add/subtract within 1,000,000
5.NBT.B.5	Multi-digit multiplication
6.NS.B.2	Multi-digit division

Domain: 5.OA Operations and Algebraic Thinking

5.OA.A Additional Cluster: Write and interpret numerical expressions.

Standard	Rigor	SMP Bundle
5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols, including expressions in which whole numbers and fractions appear.	Procedural	Problem Solving
5.OA.A.2 Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by* $\frac{1}{2}$ *as* $\frac{1}{2} \times (8 + 7)$. *Recognize that* $3 \times (\frac{18}{19} + \frac{2}{3})$ *is three times as large as* $\frac{18}{19} + \frac{2}{3}$, *without having to calculate the indicated sum or product.*	Conceptual	Communicating Reasoning

5.OA.B Additional Cluster: Analyze a pair of number sequences.

Standard	Rigor	SMP Bundle
5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane; explain informally why this is so. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence.*	Conceptual Procedural	Communicating Reasoning

Domain: 5.NBT Numbers and Operations in Base Ten

5.NBT.A Major Cluster: Understand the place value system.

Standard	Rigor	SMP Bundle
5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.	Conceptual	Communicating Reasoning
5.NBT.A.2 Explain and use patterns in the number of zeros of the product when multiplying a number by powers of 10 and use patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	Conceptual Procedural	Communicating Reasoning
5.NBT.A.3 Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. *For example,* $347.392 = 300 + 40 + 7 + 0.3 + 0.09 + 0.002$. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Conceptual Procedural	Communicating Reasoning
5.NBT.A.4 Use place value understanding to round decimals to any place. *For example,* 5.43 *rounded to the tenths is* 5.4 *because the last digit must be in the place the decimal is rounded to.* Note: 5.40 would not be correct as it is rounded to the hundredths, not tenths.	Conceptual Procedural	Communicating Reasoning

5.NBT.B Major Cluster: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Standard	Rigor	SMP Bundle
5.NBT.B.5 Fluently multiply whole multi-digit numbers including using an algorithm. Algorithms may include the standard algorithm, partial products, area model. Note: Fluency of this standard is critical by the end of grade level.	Conceptual Procedural	Problem Solving Modeling & Data Analysis
5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division, including the standard algorithm. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Conceptual Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis
5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths. Be able to illustrate and explain using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Conceptual Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: 5.NF Number and Operations–Fractions

5.NF.A Major Cluster: Use equivalent fractions as a strategy to add and subtract fractions.

Standard	Rigor	SMP Bundle
5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$. *(In general,* $\frac{a}{b}+\frac{c}{d}=\frac{(ad+bc)}{bd}$*).*	Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis
5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. *For example, by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, my friend and I each have some lemons. We need 1 cup of lemon juice to make lemonade. If I squeeze* $\frac{1}{2}$ *cup of lemon juice and my friend squeezes* $\frac{2}{5}$ *a cup of lemon juice how much lemon juice do we have? Is it enough?*	Conceptual Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

5.NF.B Major Cluster: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standard	Rigor	SMP Bundle
5.NF.B.3 Interpret that a fraction is the division of the numerator by the denominator ($\frac{a}{b} = a \div 5$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, by using visual fraction models or equations to represent the problem. *For example, if 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Conceptual Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
5.NF.B.4 Apply and extend earlier understandings of multiplication to multiply a fraction or whole number by a fraction. (This standard does not include mixed numbers) a. Interpret the product $(\frac{a}{b}) \times q$ as a part of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. Recognize that $\frac{1}{b} \times q = q \div b$ (dividing by a whole is the same as multiplying by the reciprocal. *For example, use a visual fraction model to show* $(\frac{2}{3}) \times 4 = \frac{8}{3}$, *and create a story context for this equation. Do the same with* $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. *(In general,* $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$.*)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.	Conceptual Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis
5.NF.B.5 Interpret multiplication as scaling (resizing) by: a. Comparing the size of a product to the size of one factor based on the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1.	Conceptual	Problem Solving Modeling & Data Analysis
5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers. *For example, by using visual fraction models or equations to represent the problem.*	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

Standard	Rigor	SMP Bundle
5.NF.B.7 Apply and extend earlier understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (This standard does not include dividing fractions by fractions). a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $(\frac{1}{3}) \div 4$, *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $(\frac{1}{3}) \div 4 = \frac{1}{12}$ *because* $(\frac{1}{12}) \times 4 = \frac{1}{3}$. b. Interpret division of a whole number by a unit fraction and compute such quotients. *For example, create a story context for* $4 \div (\frac{1}{5})$, *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div (\frac{1}{5}) = 20$ *because* $20 \times (\frac{1}{5}) = 4$. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions. *For example, by using visual fraction models and equations to represent the problem: how much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{1}{3}$ *cup servings are in 2 cups of raisins?*	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: 5.MD Measurement and Data

5.MD.A Supporting Cluster: Convert like measurement units within a given measurement system.

Standard	Rigor	SMP Bundle
5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system and use these conversions in solving multi-step, real world problems. *For example, (convert 5 cm to 0.05 m)*.	Procedural Application	Problem Solving Modeling & Data Analysis

5.MD.B Supporting Cluster: Represent and interpret data.

Standard	Rigor	SMP Bundle
5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Use operations on fractions to solve problems involving information presented in line plots. • $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}$ • $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ • $\frac{1}{2}, \frac{2}{2}$	Conceptual Procedural Application	Modeling & Data Analysis

5.MD.C Major Cluster: Geometric measurement: understand concepts of volume and relate volume to multiplication and addition.

Standard	Rigor	SMP Bundle
5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of n cubic units.	Conceptual	Problem Solving Modeling & Data Analysis
5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	Conceptual Procedural	Problem Solving Modeling & Data Analysis
5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be if found by multiplying the edge lengths or equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ (where B stands for the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts (composite figures), applying this technique to solve real world problems. *For example, find the volume of composite figures.*	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: 5.G Geometry

5.G.A Additional Cluster: Graph points on the coordinate plane to solve real-world and mathematical problems.

Standard	Rigor	SMP Bundle
5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Plot points in the first quadrant of a coordinate plane. Understand that the first number indicates how far to travel from the origin in the direction of the x-axis, and the second number indicates how far to travel in the direction of the y-axis, with the convention that the names of the two axes and the coordinates correspond (x, y).	Conceptual	Problem Solving Modeling & Data Analysis
5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

5.G.B Additional Cluster: Classify two-dimensional figures into categories based on their properties.

Standard	Rigor	SMP Bundle
5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	Conceptual	Communicating Reasoning
5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.	Conceptual	Communicating Reasoning

Standards for Mathematical Practice: Grades 6–8

SMP1: Make sense of problems and persevere in solving them.

Mathematically proficient middle school students set out to understand a problem and then look for entry points to its solution. They analyze problem conditions and goals, translating, for example, verbal descriptions into equations, diagrams, or graphs as part of the process. They consider analogous problems and try special cases and simpler forms of the original problem to gain insight into its solution. For example, to understand why a 20% discount followed by a 20% markup does not return an item to its original price, they might translate the situation into a tape diagram or a general equation, or they might first consider the situation for an item priced at $100. Mathematically proficient students can explain how alternate representations of problem conditions relate. For example, they can identify correspondences between the solution to a word problem that uses only arithmetic and a solution that uses variables and algebra, and they can navigate among tables, graphs, and equations representing linear relationships to gain insights into the role played by a constant rate of change. Mathematically proficient students check their approach, asking themselves, “Does this approach make sense?” and “Can I solve this problem differently?” While working on a problem, they monitor and evaluate their progress and change course if necessary. They can understand the approaches of others to solving complex problems and compare approaches.

SMP 2: Reason abstractly and quantitatively.

Mathematically proficient middle school students make sense of quantities and relationships in problem situations. For example, they can apply ratio reasoning to convert measurement units and proportional relationships to solve percentage problems. They represent problem situations using symbols and then manipulate those symbols in search of a solution (decontextualize). They can, for example, solve problems involving unit rates by representing the situations in equation form. Mathematically proficient students also pause, as needed, during problem solving to double-check the meaning of the symbols involved. In the process, they can look at the applicable units of measure to clarify or inform solution steps (contextualize). Students can integrate quantitative information and concepts expressed in text and visual formats. Quantitative reasoning also entails knowing and flexibly using different properties of operations and objects. For example, in middle school, students use properties of operations to generate equivalent expressions and use the number line to understand multiplication and division of rational numbers.

SMP 3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient middle school students understand and use assumptions, definitions, and previously established results in constructing verbal and written arguments. They make and explore the validity of conjectures. They can recognize and appreciate using counterexamples, for example, using numerical counterexamples to identify common errors in algebraic manipulation, such as thinking that $5 - 2x$ is equivalent to $3x$. Conversely, given a pair of equivalent algebraic expressions, they can show that the two expressions name the same number regardless of which value is substituted by showing which operation’s properties can be applied to transform one expression into the other. They can explain and justify their conclusions to others using numerals, symbols, and visuals; they can listen and read others’ arguments, deciding whether they make sense and asking questions to clarify the arguments. They also reason inductively about data, making plausible arguments considering the context from which the data arose. For example, they might argue that growth spurts explain the significant variability of heights in their class and that school admission policies explain the small variability of ages. Mathematically proficient students can also compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Students engage in collaborative discussions, drawing on evidence from problem texts and arguments of others, follow conventions for collegial discussions, and qualify their views in considering the evidence presented. They consider questions, such as “How did they get that?”, “Why is that true?”, and “Does that always work?”

SMP 4: Model with mathematics.

Mathematically proficient middle school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. Applying mathematics might be as simple as translating a verbal or written description to a drawing or mathematical expression. It might entail using the mathematics of proportional relationships to plan a school event or using data to analyze a problem in the community. Mathematically proficient students are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. For example, they can roughly fit a line to a scatter plot to make predictions and gather experimental data to approximate a probability. They can identify important quantities in each relationship, such as rates of change, and represent situations using such tools as diagrams, tables, graphs, flow charts, and formulas. They can analyze their representations mathematically, use the results in the context of the situation, and then reflect on whether the results make sense while possibly improving the model.

Note: Although one can use physical objects and drawings to model a situation, using these tools absent a contextual situation does not exemplify practice standard SMP4. For example, drawing an area model to illustrate the distributive property in $4(t + s) = 4t + 4s$ would not be an example of practice standard of SMP4. SMP4 is about applying math to a problem in context.

SMP 5: Use appropriate tools strategically.

Mathematically proficient middle school students strategically consider the available tools when solving a mathematical problem and exploring a mathematical relationship. These tools include pencil and paper, concrete models, rulers, protractors, graphing calculators, spreadsheets, statistical packages, or dynamic geometry software. Proficient students recognize the insights to be gained and the limitations of these tools, making sound decisions about when they might be helpful. For example, they use estimation to check reasonableness, graph functions designed by expressions to picture the way one quantity depends on another, use algebra tiles to see how the properties of operations familiar from the elementary grades continue to apply to algebraic expressions, use graphing calculators to approximate solutions to systems of equations, use spreadsheets to analyze data sets of realistic size or use dynamic geometry software to discover properties of parallelograms. Students are also strategic about when not to use tools, such as simplifying an expression before substituting values (SMP 7), rounding the inputs to a calculation, and calculating on paper when an approximate answer is enough (SMP 6). When making mathematical models, students know that technology can enable them to visualize the results of their assumptions, explore consequences, and compare predictions with data. Mathematically proficient students can identify relevant external mathematical resources, such as digital content found on a website, and use them to pose or solve problems.

SMP 6: Attend to precision.

Mathematically proficient middle school students communicate precisely with others verbally and in writing. They represent claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound and valid reasoning, well-chosen details, and precise language. They use clear definitions in discussions with others and their reasoning and determine the meaning of symbols, terms, and phrases used in specific mathematical contexts. For example, they can use the definition of rational numbers to explain why a number is irrational and describe congruence and similarity in terms of transformations in the plane. They state the meaning of the symbols they choose consistently and appropriately, such as inputs and outputs represented by function notation. They are careful about specifying units of measure and label axes to display the correct correspondence between quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate to the context. For example, they accurately apply scientific notation to large numbers and use measures of center to describe data sets. Diligence and attention to detail are mathematical virtues: mathematically proficient students care that an answer is correct; they check their work; they solve the problem another way; they demonstrate accountability by identifying, correcting, and learning from their mistakes.

SMP 7: Look for and make use of structure.

Mathematically proficient middle school students look closely to discern a pattern or structure. They might use a structure of the number line to demonstrate that the distance between two rational numbers is the absolute value of their difference, find the relationship between slopes and solution sets of systems of linear equations, and see that the equation $3x = 2y$ represents a proportional relationship with a unit rate of $\frac{3}{2} = 1.5$. They might recognize how to use the Pythagorean Theorem to find distances between points in the coordinate plane and identify right triangles that can be used to find the length of a diagonal in a rectangular prism. They can also step back for an overview and shift perspective, as in finding a representation of consecutive numbers that shows all sums of three consecutive whole numbers are divisible by six. They can see complicated things as single objects, such as seeing two successive reflections across parallel lines as a translation along a line perpendicular to the parallel lines or understanding $1.05a$ as an original value, a, plus 5% of that value, $0.05a$.

SMP 8: Look for and express regularity in repeated reasoning.

Mathematically proficient middle school students notice if calculations are repeated and look for general methods and shortcuts. Working with tables of equivalent ratios, they might deduce the corresponding multiplicative relationships and generalize the relationship to rates. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope equal to 3, students might abstract the equation $\frac{y-2}{x-1}$ = 3. Noticing the regularity with each interior angle, the sum increases with the number of sides in a polygon might lead them to the general formula for the interior angle sum of an *n* -gon. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their immediate results.

(adapted from Iowa Mathematics Standards, 2010)

Sixth Grade

In Grade 6, instructional time should focus on three major areas:

1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.

   In grade 6, students take a significant step from the work they did in earlier grades of multiplying and dividing with quantities in word problems to using reasoning about multiplication and division in situations in which two variable quantities vary together in a proportional relationship. Thus, students expand the scope of situations in which they can use multiplication and division to solve problems. Students connect their understanding of multiplication and division with ratios and rates, and they connect ratios and fractions. They construct and analyze tables, such as tables of quantities in equivalent ratios, and use equations (such as $3x = y$) to describe relationships.

2. Completing understanding of the division of fractions and extending the notion of numbers to the system of rational numbers, which includes negative numbers.

   Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why procedures for dividing fractions make sense, and they divide fractions to solve problems. Students extend earlier understandings of numbers to the entire rational number system, including negative rational numbers. They reason about the order and absolute value of rational numbers and the location of points in all four coordinate plane quadrants.

3. Writing, interpreting, and using expressions and equations.

   Students understand the use of variables in mathematical expressions and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use properties of operations to rewrite expressions in equivalent forms. Students know that solutions of an equation are values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple equations.

Grade 6 Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Ratios and Proportional Relationships (RP)

- **Apply ratio concepts and use ratio reasoning to solve problems.**

The Number System (NS)

- **Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**
- Compute with multi-digit numbers and find common factors and multiples.
- **Apply and extend previous understandings of numbers to the system of rational numbers.**

Expressions and Equations (EE)

- **Apply and extend previous understandings of arithmetic to algebraic expressions.**
- **Reason about and solve one-variable equations and inequalities.**
- **Represent and analyze quantitative relationships between dependent and independent variables.**

Geometry (G)

- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability (SP)

- Develop understanding of statistical variability.
- Summarize and describe distributions.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

End of Grade Level Fluency Table

Standard	Fluencies
K.OA.A.5	Add/subtract within 5
1.OA.C.6	Add/subtract within 10
2.OA.B.2	Add and subtract within 20
2.NBT.B.5	Add/subtract within 100
3.OA.C.7	Multiply/divide within 100
3.NBT.A.2	Add/subtract within 1,000
4.NBT.B.4	Add/subtract within 1,000,000
5.NBT.B.5	Multi-digit multiplication
6.NS.B.2	Multi-digit division
6.NS.B.3	Multi-digit decimal operations

Domain: 6.RP Ratios and Proportional Relationships

6.RP.A Major Cluster: Apply ratio concepts and use ratio reasoning to solve problems.

Standard	Rigor	SMP Bundle
6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was* $2{:}1$*, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes. "*	Conceptual	Problem Solving Modeling & Data Analysis
6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio *a*:*b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." Expectations for unit rates in this grade are limited to non-complex fractions.*	Conceptual	Problem Solving Modeling & Data Analysis
6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Use unit rates and scaling to solve problems about proportional relationships, including problems involving unit pricing and constant speed. c. Find a percentage of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percentage. *For example,* 30% *of a quantity means* $\frac{30}{100}$ *times the quantity.* d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: 6.NS The Number System

6.NS.A Major Cluster: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Standard	Rigor	SMP Bundle
6.NS.A.1 Use and interpret models to compute quotients of fractions. Solve word problems involving division of fractions by fractions. Be able to use visual fraction models and equations to represent the problem. *For example, create a story context for* $(\frac{2}{3}) \div (\frac{3}{4})$ *and use a visual fraction - model to show the quotient; use the relationship between multiplication and division to explain that* $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ *because* $\frac{3}{4}$ *of* $\frac{8}{9}$ *is* $\frac{2}{3}$*. (In general,* $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$*). If* $\frac{2}{3}$ *of a shoelace is* $\frac{1}{2}$ *meter long, how many meters long is the shoelace? How many* $\frac{3}{4}$ *cup servings are in* $\frac{2}{3}$ *of a cup of yogurt? How wide is a rectangular strip of land with length* $\frac{3}{4}$ *mi and area* $\frac{1}{2}$ *square-mile?*	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

6.NS.B Additional Cluster: Compute with multi-digit numbers and find common factors and multiples.

Standard	Rigor	SMP Bundle
6.NS.B.2 Divide multi-digit numbers using the standard algorithm. For at least 4 digits by 1-digit division by hand; more complicated cases using technology. *For example,* $\frac{6{,}389}{7}$.	Procedural	Problem Solving
6.NS.B.3 Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. For more complex cases, use technology.	Procedural	Problem Solving
6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express* $36 + 8$ *as* $4\,(9 + 2)$.	Conceptual Procedural	Problem Solving

6.NS.C Major Cluster: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard	Rigor	SMP Bundle
6.NS.C.5 Describe quantities having opposite directions or values using positive and negative numbers: temperature above/below zero, elevation above/below sea level, credits/debits, and positive/negative electric charge. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	Conceptual Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
6.NS.C.6 Represent a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from earlier grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself. *For example,* $-(-3) = 3$*, and that 0 is its own opposite.* b. Describe locations in the coordinate plane using signed numbers in ordered pairs; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	Conceptual	Problem Solving Communicating Reasoning Modeling & Data Analysis
6.NS.C.7 Compare, order and describe the absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret* $-3 > -7$ *as a statement that* -3 *is located to the right of* -7 *on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write* $-3°C > -7°C$ *to express the fact that* $-3°\,C$ *is warmer than* $-7°C$*.* *c.* Describe the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of* -30 *dollars, write* $\|-30\| = 30$ *to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than* -30 *dollars represents a debt greater than* 30*30 dollars.*	Conceptual	Problem Solving Modeling & Data Analysis
6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

Domain: 6.EE Expressions and Equations

6.EE.A Major Cluster: Apply and extend previous understandings of arithmetic to algebraic expressions.

Standard	Rigor	SMP Bundle
6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.	Conceptual Procedural	Problem Solving
6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* $5 - y$. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* $2(8 + 7)$ *as a product of two factors; view* $(8 + 7)$ *as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = \frac{1}{2}$.	Procedural Application	Problem Solving Modeling & Data Analysis
6.EE.A.3 Apply the properties of operations to generate equivalent expressions. Know that expressions are called equivalent when they name the same number regardless of which value is substituted into them. *For example, apply the distributive property to the expression* $3(2 + x)$ *to produce the equivalent expression* $6 + 3x$*; apply the distributive property to the expression* $24x + 8y$ *to produce the equivalent expression* $6(4x + 3y)$*; apply properties of operations to* $y + y + y$ *to produce the equivalent expression* $3y$.	Procedural Application	Problem Solving Communicating Reasoning
6.EE.A.4 Describe the properties of operations used to show two expressions are equivalent. *For example, show that* $3c + 3cd$ *and* $3c(1 + d)$ *are equivalent.*	Conceptual	Problem Solving Communicating Reasoning

6.EE.B Major Cluster: Reason about and solve one-variable equations and inequalities.

Standard	Rigor	SMP Bundle
6.EE.B.5 Use substitution to determine whether a given number in a specified set makes an equation or inequality true. Solving an equation or inequality is a process of answering a question: Which values from a specified set, if any, make the equation or inequality true?	Conceptual Procedural	Problem Solving Modeling & Data Analysis
6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or depending on the purpose at hand, any number in a specified set.	Conceptual Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	Procedural Application	Problem Solving Modeling & Data Analysis
6.EE.B.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that an inequality of the form $x > c$ or $x < c$ has infinitely many solutions; use a number line diagram to represent infinitely many solutions of such an inequality.	Conceptual Application	Problem Solving Modeling & Data Analysis

6.EE.C Major Cluster: Represent and analyze quantitative relationships between dependent and independent variables.

Standard	Rigor	SMP Bundle
6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity in terms of the other quantity. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.*	Conceptual Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: 6.G Geometry

6.G.A Supporting Cluster: Solve real-world and mathematical problems involving area, surface area, and volume.

Standard	Rigor	SMP Bundle
6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	Procedural Application	Problem Solving Modeling & Data Analysis
6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ (where B stands for the area of the base) to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis
6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	Procedural Application	Problem Solving Modeling & Data Analysis
6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

Domain: 6.SP Statistics and Probability

6.SP.A Additional Cluster: Develop understanding of statistical variability.

Standard	Rigor	SMP Bundle
6.SP.A.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	Conceptual	Modeling & Data Analysis
6.SP.A.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	Conceptual	Modeling & Data Analysis
6.SP.A.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	Conceptual	Modeling & Data Analysis

6.SP.B Additional Cluster: Summarize and describe distributions.

Standard	Rigor	SMP Bundle
6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	Conceptual Procedural Application	Modeling & Data Analysis
6.SP.B.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	Conceptual Application	Modeling & Data Analysis

Seventh Grade

Narrative

In grade 7, instructional time should focus on three major areas:

1. Developing an understanding of and applying proportional relationships.

   In grade 7, students deepen their understanding and skill in proportional relationships. They use their ratios, rates, and proportionality knowledge to solve various percent problems, including discounts, interest, taxes, tips, and percent increase or decrease. These are real-life scenarios where mathematics is applied. This skill is helpful in various fields, such as economics and physics. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that they preserve relationships of lengths within an object in similar objects. Identifying proportional relationships are a fundamental skill in architecture and engineering.

2. Developing an understanding of operations with rational numbers and working with expressions and linear equations.

   Students unify their understanding of numbers by recognizing fractions, decimals (with a finite or a repeating decimal representation), and percentages as different representations of rational numbers. Unification is a significant step in their mathematical journey. They extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations. By applying these properties and viewing negative numbers in everyday contexts (for example, amounts owed or temperatures below zero), students explain and interpret the generalizations for adding, subtracting, multiplying, and dividing with negative numbers. Operations with negative numbers are a complex concept that students are now ready to tackle.

3. Writing, interpreting, and using expressions and equations.

   Students extend their earlier understanding of the use of variables in mathematical expressions and formulas to solve problems. They are honing the crucial skill of problem-solving which will be invaluable in future math courses courses and careers such as engineering and computer science. Students deepen their understanding that expressions in different forms can be equivalent, and they use properties of operations to rewrite expressions in equivalent forms. This is a skill that will help them in higher-level math. Students know that solutions to an inequality are values of the variables that make the inequality true. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. This is a practical application of their learning and a testament to their problem-solving abilities.

Grade 7 Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Ratios and Proportional Relationships (RP)

- **Analyze proportional relationships and use them to solve real-world and mathematical problems.**

The Number System (NS)

- **Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**

Expressions and Equations (EE)

- **Use properties of operations to generate equivalent expressions.**
- **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

Geometry (G)

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability (SP)

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate and model chance processes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: 7.RP Ratios and Proportional Relationships

7.RP.A Major Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standard	Rigor	SMP Bundle
7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks* $\frac{1}{2}$ *mile in each* $\frac{1}{4}$ *hour, compute the unit rate as the complex fraction* $\frac{\frac{1}{2}}{\frac{1}{4}}$ *miles per hour, equivalently 2 miles per hour. Give a reason it is a better value to buy a supply of an item at a cost of $22.50 for ten pounds than at a cost of $1.50 for* $\frac{1}{2}$ *pound.*	Procedural Application	Problem Solving Modeling & Data Analysis
7.RP.A.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship. *For example, by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.* b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items bought at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$ where r is the unit rate.	Conceptual Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *For example: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	Procedural Application	Problem Solving Modeling & Data Analysis

Domain: 7.NS The Number System

7.NS.A Major Cluster: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Standard	Rigor	SMP Bundle
7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Use a model to describe $p+q$ as a number located a distance $\lvert q \rvert$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Use a model to describe subtraction of rational numbers as adding the additive inverse, $p-q=p+(-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
7.NS.A.2 Apply and extend earlier understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Use properties of operations, particularly the distributive property, leading to generalizations for products such as $(-1)(-1)=1$ for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Use properties of operations, particularly the distributive property, leading to generalizations for quotients of integers (provided that the divisor is not zero). If p and q are integers, then $-\left(\frac{p}{q}\right)=\frac{(-p)}{q}=\frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts. c. Multiply and divide rational numbers. d. Convert a rational number to a decimal; know that the decimal form of a rational number terminates in 0s or eventually repeats.	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis
7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions, a fraction within a fraction.	Procedural Application	Problem Solving Modeling & Data Analysis

Domain: 7.EE Expressions and Equations

7.EE.A Major Cluster: Use properties of operations to generate equivalent expressions.

Standard	Rigor	SMP Bundle
7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	Conceptual Procedural	Problem Solving Communicating Reasoning
7.EE.A.2 Describe how rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* $a + 0.05a = 1.05a$ *means that "increase by* 5%*" is the same as "multiply by* 1.05*."*	Conceptual	Problem Solving Communicating Reasoning

7.EE.B Major Cluster: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Standard	Rigor	SMP Bundle
7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example***,** *If someone making $25 an hour gets a* 10% *raise, that is an additional* $\frac{1}{10}$ *of their salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar* $9\ \frac{3}{4}$ *inches long in the center of a door that is* $27\ \frac{1}{2}$ *inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	Procedural Application	Problem Solving Modeling & Data Analysis
7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions.* $px + q > r$	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

Domain: 7.G Geometry

7.G.A Additional Cluster: Draw, construct, and describe geometrical figures and describe the relationships between them.

Standard	Rigor	SMP Bundle
7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	Procedural Application	Problem Solving Modeling & Data Analysis
7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	Conceptual Procedural	Problem Solving Modeling & Data Analysis
7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	Conceptual	Problem Solving Modeling & Data Analysis

7.G.B Additional Cluster: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Standard	Rigor	SMP Bundle
7.G.B.4 Choose the formula needed and use it to solve problems involving the area and circumference of a circle. *For example, a 15.1 in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?*	Conceptual Procedural	Problem Solving Modeling & Data Analysis
7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	Conceptual Procedural	Problem Solving Modeling & Data Analysis
7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Procedural Application	Problem Solving Modeling & Data Analysis

Domain: 7.SP Statistics and Probability

7.SP.A Supporting Cluster: Use random sampling to draw inferences about a population.

Standard	Rigor	SMP Bundle
7.SP.A.1 Describe how statistics can be used to gain information about a population by examining a sample of the population, recognizing that generalizations about a population from a sample are valid only if the sample is representative of that population. Explain that random sampling tends to produce representative samples and support valid inferences.	Conceptual	Modeling & Data Analysis
7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data and observe the variation in predictions across multiple surveys.*	Conceptual Application	Modeling & Data Analysis

7.SP.B Additional Cluster: Draw informal comparative inferences about two populations.

Standard	Rigor	SMP Bundle
7.SP.B.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	Conceptual Application	Modeling & Data Analysis
7.SP.B.4 Use measures of center (*for example, mode, median, mean*) and measures of variability (*for example, range, interquartile range, mean absolute deviation*) for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	Conceptual Application	Modeling & Data Analysis

7.SP.C Supporting Cluster: Investigate and model chance processes.

Standard	Rigor	SMP Bundle
7.SP.C.5 Describe the probability of a chance event as a number between 0 and 1 that expresses the likelihood of the event occurring. (*for example, larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around* $\frac{1}{2}$ *indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event*).	Conceptual	Modeling & Data Analysis
7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency. Given the probability of a chance event, predict the approximate relative frequency that will be observed, and collect data to assess the agreement between the probability and the observed frequency. *For example, collect data to approximate the probability that a tossed paper cup will land open-end down. Your friend calculated that the probability of "rolling double sixes" with a pair of number cubes is* $\frac{1}{36}$ *(which is the wrong answer) collect data to see how well this probability agrees with the observation frequency.*	Conceptual Application	Modeling & Data Analysis
7.SP.C.7 Calculate probabilities of simple events under an assumption of equal probability for all outcomes. *For example, suppose that one student in seventh grade will be chosen to speak at a school assembly. On the assumption that every student is equally likely to be chosen, calculate the probability that the youngest seventh grader will be chosen and the probability that a member of Homeroom 701 will be chosen. Calculate the probability of a spinner landing on a certain color, assuming that all of the colors are equally likely outcomes.*	Conceptual Application	Modeling & Data Analysis
7.SP.C.8 Calculate probabilities of compound events using organized lists, tables, tree diagrams, and simulation. *For example, Calculate the probability of "rolling double sixes." Use a simulation to approximate the answer to the question. For example, if* 40% *of blood donors have type A blood, what is the probability that it will take at least 4 blood donors to find one with type A blood?*	Procedural Application	Modeling & Data Analysis

Eighth Grade

In grade 8, instructional time should focus on three major areas:

1. Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation and solving linear equations and systems of linear equations.

   In grade 8, students apply the properties of integer exponents to generate equivalent numerical expressions. Students use square root, cube root to evaluate square roots of small perfect squares and cube roots of small perfect cubes. They learn scientific notation and use it in real world contexts.

   Students use linear equations and systems of linear equations to represent, analyze, and solve various real-world problems. They recognize equations for proportional relationships ($\frac{y}{x} = m$ or $y = \frac{m}{x}$) as special linear equations, understanding that the constant of proportionality (m) is the slope, and the graph is a straight line through the origin. They know that the slope (m) of the graph of a linear function is a constant rate of change, so if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m * A$. Students also use a linear equation to model the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model and assessing its fit to the data are done informally. Interpreting the model requires students to express a relationship between the two quantities and to interpret components of the relationship (such as slope and y -intercept) in terms of the situation.

   Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of the slope of a line to analyze situations and solve problems. These concepts and skills become more refined and efficient in Algebra 1 as they learn more strategies to use with equations and systems that require manipulation.

2. Grasping the concept of a function and using functions to describe quantitative relationships.

   Students grasp the concept of function as a rule that assigns exactly one output to each input. They understand that functions describe situations where one quantity determines another. They translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function reflect in the different representations. Students extend their understanding of functions in Algebra 1 when they begin using function notation and develop a sense of domain and range for linear and nonlinear functions.

3. Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

   Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and solve problems. Students demonstrate that the angles in a triangle sum to a straight-line angle, and that various line configurations produce similar triangles due to transversal line intersections with parallel lines. Students explain at least one proof of the Pythagorean Theorem and at least one proof of its converse. To demonstrate its practical use in various scenarios, students apply the Pythagorean Theorem to find distances between points on the coordinate plane, find lengths, and analyze polygons.

Grade 8 Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

The Number System (NS)

- Work with numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations (EE)

- **Work with radicals and integer exponents.**
- **Understand the connections between proportional relationships, lines, and linear equations.**
- **Analyze and solve linear equations and pairs of simultaneous linear equations**

Functions (F)

- **Define, evaluate, and compare functions.**
- **Use functions to model relationships between quantities.**

Geometry (G))

- **Demonstrate congruence and similarity using physical models, patty paper or geometry software.**
- **Explain and apply the Pythagorean Theorem.**
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Statistics and Probability (SP)

- Investigate patterns of association in bivariate data.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: 8.NS The Number System

8.NS.A Supporting Cluster: Work with numbers that are not rational, and approximate them by rational numbers.

Standard	Rigor	SMP Bundle
8.NS.A.1 Classify and explain numbers as rational or irrational. For rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.	Conceptual Procedural	Communicating Reasoning
8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions. *For example, estimate the value of* $\sqrt{2}$*. By truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue - on to get better approximations.*	Conceptual Procedural	Communicating Reasoning

Domain: 8.EE Expressions and Equations

8.EE.A Major Cluster: Work with radicals and integer exponents.

Standard	Rigor	SMP Bundle
8.EE.A.1 Apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.	Procedural	Problem Solving Communicating Reasoning
8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Students evaluate square roots of small perfect squares and cube roots of small perfect cubes. Use bases 1 through 5 and 10 for cubes.	Conceptual Procedural	Problem Solving Communicating Reasoning
8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate exceptionally large or small quantities and to express how many times as much one is than another. *For example, estimate the population of the United States as 3 times* 10^8 *and the population of the world as 7 times* 10^9*, and determine that the world population is more than 20 times larger.*	Conceptual Application	Problem Solving Modeling & Data Analysis
8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of convenient size for quantities. *For example, use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.*	Conceptual Procedural	Problem Solving Modeling & Data Analysis

8.EE.B Major Cluster: Understand the connections between proportional relationships, lines, and linear equations.

Standard	Rigor	SMP Bundle
8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.	Conceptual Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis
8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. a. Derive from this principle the equation $y = mx$ for a line through the origin. b. Derive from this principle the equation $y = mx + b$ for a line intercepting the vertical axis at b.	Conceptual Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis

8.EE.C Major Cluster: Analyze and solve linear equations and pairs of simultaneous linear equations.

Standard	Rigor	SMP Bundle
8.EE.C.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Standard	Rigor	SMP Bundle
8.EE.C.8 Analyze and solve pairs of simultaneous linear equations. a. Describe how the solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple (by inspection) cases. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6;* $x - y = 11$ *and* $2x + y = 19$. c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: 8.F Functions

8.F.A Major Cluster: Define, evaluate, and compare functions.

Standard	Rigor	SMP Bundle
8.F.A.1 Describe a function as a rule that assigns to each input exactly one output and the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in grade 8.	Conceptual	Problem Solving Communicating Reasoning
8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	Conceptual	Problem Solving Communicating Reasoning
8.F.A.3 Interpret the equation $y = mx + b$ as defining a function that assigns to each input value x the output value $mx + b$; this is a linear function whose graph is a straight line. Give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points* $(1,1)$, $(2,4)$ *and* $(3,9)$, *which are not on a straight line.*	Conceptual	Problem Solving Communicating Reasoning

8.F.B Major Cluster: Use functions to model relationships between quantities.

Standard	Rigor	SMP Bundle
8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.	Conceptual Application	Problem Solving Modeling & Data Analysis
8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph. *For example, identify where the function is increasing or decreasing, and if it is linear or nonlinear. Sketch a graph that shows the qualitative features of a function that has been described verbally.*	Conceptual Application	Problem Solving Modeling & Data Analysis

Domain: 8.G Geometry

8.G.A Major Cluster: Demonstrate congruence and similarity using physical models, patty paper or geometry software.

Standard	Rigor	SMP Bundle
8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	Conceptual	Problem Solving Communicating Reasoning
8.G.A.2 Explain that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Given two congruent figures, describe a sequence of rigid transformations that proves the congruence between them.	Conceptual	Problem Solving Communicating Reasoning
8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Conceptual	Problem Solving
8.G.A.4 Explain that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that demonstrates the similarity between them.	Conceptual	Problem Solving Communicating Reasoning
8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	Procedural Application	Problem Solving Communicating Reasoning

8.G.B Major Cluster: Explain and apply the Pythagorean Theorem.

Standard	Rigor	SMP Bundle
8.G.B.6 Explain a proof of the Pythagorean Theorem and a proof of its converse.	Conceptual	Problem Solving Communicating Reasoning Modeling & Data Analysis
8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two- and three-dimensions.	Procedural Application	Problem Solving Modeling & Data Analysis
8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

8.G.C Additional Cluster: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Standard	Rigor	SMP Bundle
8.G.C.9 Apply the formulas for the volume of cones, cylinders, and spheres to solve real-world and mathematical problems.	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

Domain: 8.SP Statistics and Probability

8.SP.A Supporting Cluster: Investigate patterns of association in bivariate data.

Standard	Rigor	SMP Bundle
8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Conceptual Procedural	Modeling & Data Analysis
8.SP.A.2 For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	Conceptual	Modeling & Data Analysis
8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5* $\frac{cm}{hr}$ *has meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	Conceptual Application	Modeling & Data Analysis
8.SP.A.4 Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	Conceptual Application	Modeling & Data Analysis

Standards of Mathematical Practice: High School

SMP 1: Make sense of problems and persevere in solving them.

Mathematically proficient high school students analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. While following the solution plan, they continually ask themselves, “Does this make sense?” They check and evaluate their progress and change course if necessary. They consider analogous problems and try special cases and simpler forms of the original problem to gain insight into its solution. Depending on the context of the situation, high school students might transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs, draw diagrams of key features and relationships, graph and interpret data representations, and search for regularity or trends. Mathematically proficient students gain deeper insight into problems by using a different approach, understanding the strategies of others to solve complex problems, and identifying correspondences between different approaches.

SMP 2: Reason abstractly and quantitatively.

Mathematically proficient high school students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* - to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and the ability to *contextualize*, to pause as needed during the manipulation process to probe into the referents for the symbols involved. Students can write explanatory texts that convey their mathematical analyses and thinking, using relevant and sufficient facts, concrete details, quotations, and coherent discussion of ideas. Students can evaluate multiple sources of information presented in diverse formats (and media) to address a question or solve a problem. Quantitative reasoning entails creating a coherent representation of the situation, considering the units involved, attending to the meanings of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

SMP 3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient high school students understand and use stated assumptions, definitions, and previously established results in constructing verbal and written arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They can analyze situations by breaking them into cases and recognizing and using counterexamples and specific textual evidence to form their arguments. They justify their conclusions, communicate them, and respond to others' arguments. They reason inductively about data, making plausible arguments considering the context from which the data arose. Mathematically proficient students can also compare the effectiveness of two convincing arguments, distinguish correct logic or reasoning from that which is flawed, and if there is a flaw in an argument explain what it is and why. They can construct formal arguments relevant to specific contexts and tasks. High school students learn to determine domains to which an argument applies. Students listen or read the arguments of others, decide whether they make sense, and ask helpful questions to clarify or improve the arguments. Students engage in collaborative discussions, drawing on evidence from problem texts and arguments of others, follow conventions for collegial discussions, and qualify their views considering the evidence presented.

SMP 4: Model with mathematics.

Mathematically proficient high school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or build a function to describe how one quantity depends on another. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing they may need revision later. They can identify important quantities practically and map their relationships using diagrams, two-way tables, graphs, flow charts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has yet to serve its purpose. They can carry out all phases of the modeling cycle. Mathematically proficient high school students also retain the widely applicable techniques they first learned in middle school, such as proportional relationships, rates, and percentages. They apply these techniques as needed to actual world tasks of a complexity appropriate to high school.

SMP 5: Use appropriate tools strategically.

Mathematically proficient high school students consider the available mathematical problem-solving tools. These tools might include pencils and paper, concrete models, a ruler, a protractor, algebra tiles, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students, familiar with high school-appropriate tools, make sound decisions about when to use them, recognizing the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator; they also know how to sketch graphs of standard functions, choosing the approach over a graphing calculator when a sketch will suffice (SMP 6). They detect possible errors by strategically using estimation and other mathematical knowledge, for example, anticipating the general appearance of a function's graph by appreciating the structure of its defining expression (SMP 7). They can use software or websites to quickly generate data displays that would otherwise be time-consuming to construct by hand (such as histograms or box plots). Students use technological tools to explore and deepen their understanding of mathematical concepts and analyze realistic data sets. When making mathematical models, students know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students can identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.

SMP 6: Attend to precision.

Mathematically proficient high school students communicate precisely with others verbally and in writing, adapting their communication to specific contexts, audiences, and purposes. They increasingly use precise language not only as a mechanism for effective communication but also as a tool for understanding and solving problems. Describing their ideas precisely helps students understand the concepts in new ways. They use clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols that they choose. They are careful about specifying units of measure, labeling axes, defining terms and variables, and calculating accurately and efficiently with a degree of precision appropriate for the problem context. They present logical claims and counterclaims fairly and thoroughly in a way that anticipates the audience's knowledge, concerns, and possible biases. High school students draw specific evidence from informational sources to support analysis, reflection, and research. They critically evaluate the claims, evidence, and reasoning of others and attend to important distinctions with their claims or inconsistencies in competing claims. Students assess the conjectures... in other sources" to "Students assess texts including quantitative elements by comparing the conjectures and claims, data, analysis, and conclusions with those found in other sources. Diligence and attention to detail are mathematical virtues: mathematically proficient students care that an answer is correct, minimize errors by keeping a long calculation organized, check their work, solve the problem another way, take responsibility for careless mistakes, and correct them.

SMP 7: Look for and make use of structure.

Mathematically proficient high school students look closely to discern a pattern or structure. In the expression, $x^2 + 9x + 14$, high school students can see the 14 as $2 \times 7$and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number, times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. Students use the structure for a purpose by applying the conclusion $5 - 3(x - y)^2 \leq 5$ in the context of an applied optimization problem.

SMP 8: Look for and express regularity in repeated reasoning.

Mathematically proficient high school students notice if calculations are repeated and look for general methods and shortcuts. Noticing the regularity in the way terms sum to zero when expanding $(x-1)\,(x+1)$, $(x-1)\,(x^2+x+1)$, and $(x-1)\,(x^3+\ x^2+x+1)$ might lead students to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details and continually evaluating the reasonableness of their intermediate results.

(adapted from Iowa Mathematics Standards, 2010)

High School Course Pathways

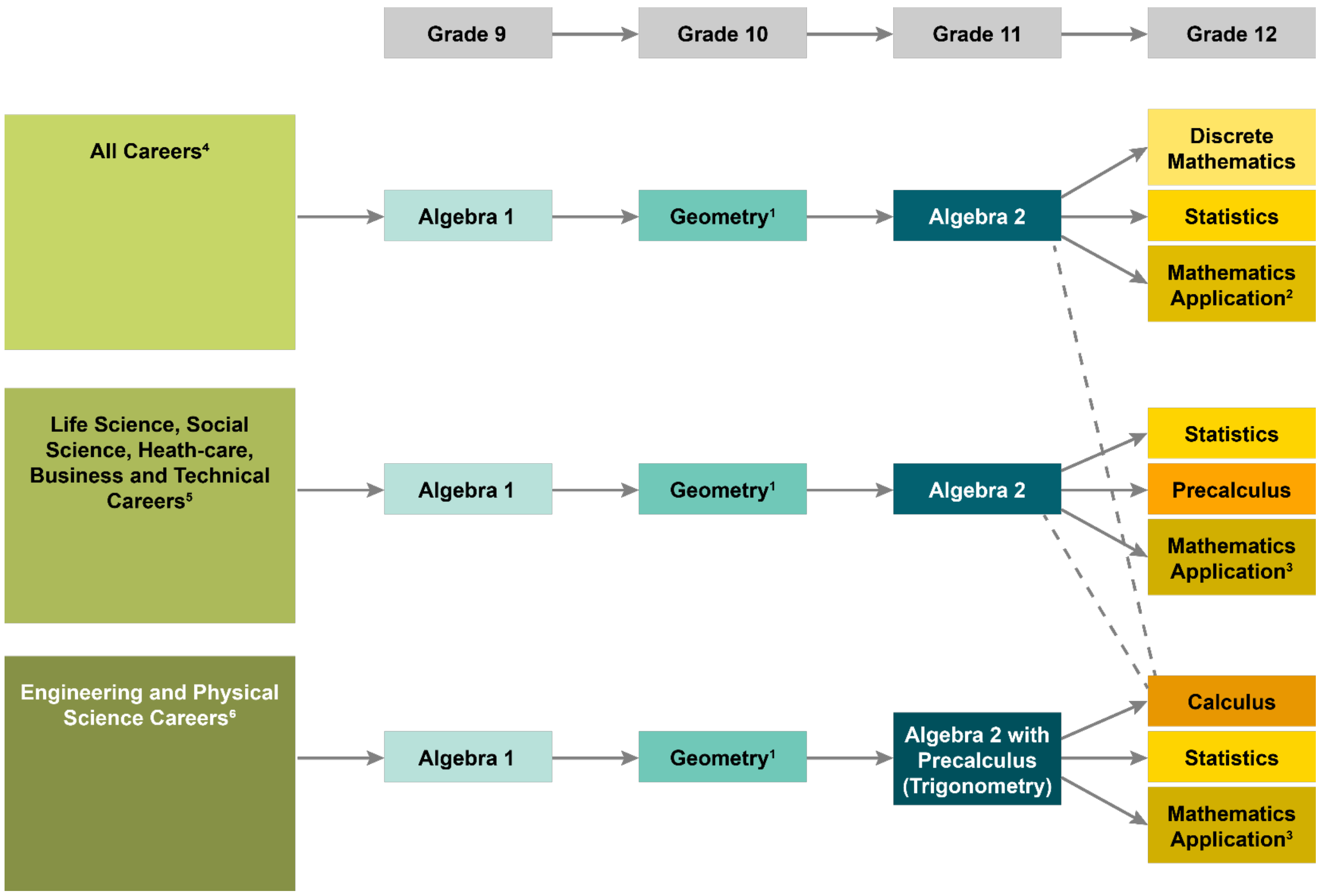
In the dynamic landscape of education and the ever-evolving demands of the professional world, achieving critical skills has become vital for students striving toward college and career readiness. Among these foundational skills, the mastery of Algebra 1 content stands out as an **indispensable cornerstone**. Algebra 1 content not only serves as a gateway to advanced mathematical concepts but also plays a pivotal role in shaping the cognitive abilities and problem-solving skills that are fundamental for success in higher education and diverse career paths.

Algebra 1 content acts as a catalyst for academic growth, fostering critical thinking and analytical reasoning. By exploring algebraic principles, students develop a deep understanding of mathematical relationships, enabling them to connect abstract concepts and real-world applications. This analytical mindset cultivated in Algebra 1 content empowers students to excel in later math courses and equips them with a versatile set of problem-solving tools applicable across various academic disciplines and professional endeavors. Educators play a crucial role in this empowerment, helping students unlock their potential and succeed academically.

As districts plan high school course sequences, it is crucial to ensure pathways exist that allow students early access to Algebra 1 content. This early access creates opportunities for students to engage with higher-level mathematics courses during high school. Educator decisions can open doors for students, potentially even considering an accelerated path that offers Algebra 1 content in eighth grade where appropriate.

Model high school course pathways are not mandated; they are models. Integrated I, II, and III content can be substituted for Algebra 1, Geometry, and Algebra 2 content. School districts have local control over bundling required standards into courses, sequences, and pathways. This flexibility empowers school districts to tailor the curriculum to meet students' needs best. However, students must take all the required standards to ensure they have had an opportunity to learn the required standards, which are the minimum level necessary for college and career readiness.

Model High School Course Pathways



- The dashed line represents where a Trigonometry or Precalculus course may be needed to ensure the student's success in Calculus.
- Integrated I, II, and III can be substituted for Algebra 1, Geometry, and Algebra 2 content.

1 The standards for Geometry include some algebraic problems that reinforce geometry and strengthen continuity in the pathways.

2 For example, Mathematics in Trade/Careers, Financial Algebra, etc.

3 For example, Data Science, Advanced Mathematical Modeling, Discrete Mathematics, etc.

4 Students on the All Careers can decide to change Pathways by taking Calculus during the senior year (dashed arrow); this requires an appropriate summer or semester bridge course.

5 Students on the pathway to Life Science, Social Science, Healthcare, Business and Technical Careers can decide to change Pathways by taking Calculus during the senior year (dashed arrow); this requires an appropriate summer or semester bridge course.

6 Students on the Engineering and Physical Science Careers can decide to change Pathways by taking statistics and or mathematics applications during their senior year instead of taking calculus.

Conceptual Categories for High School

The high school standards specify the mathematics that all students should study to be college and career ready. The required standards are included in the course sequence of Algebra 1, Geometry, and Algebra 2.

Making mathematical models is a SMP and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

The high school standards are listed in the following conceptual categories:

- Modeling (★)
- Number and Quantity (N)
- Algebra (A)
- Functions (F)
- Geometry (G)
- Statistics and Probability (S)★

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses several traditional course boundaries, potentially up through and including calculus.

Descriptions for each conceptual category is listed on the next pages.

Mathematics | High School—Modeling ★

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be quite simple, such as writing total cost as a product of unit price and number bought or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is a creative process. Like every such process, this depends on acquired expertise as well as creativity.

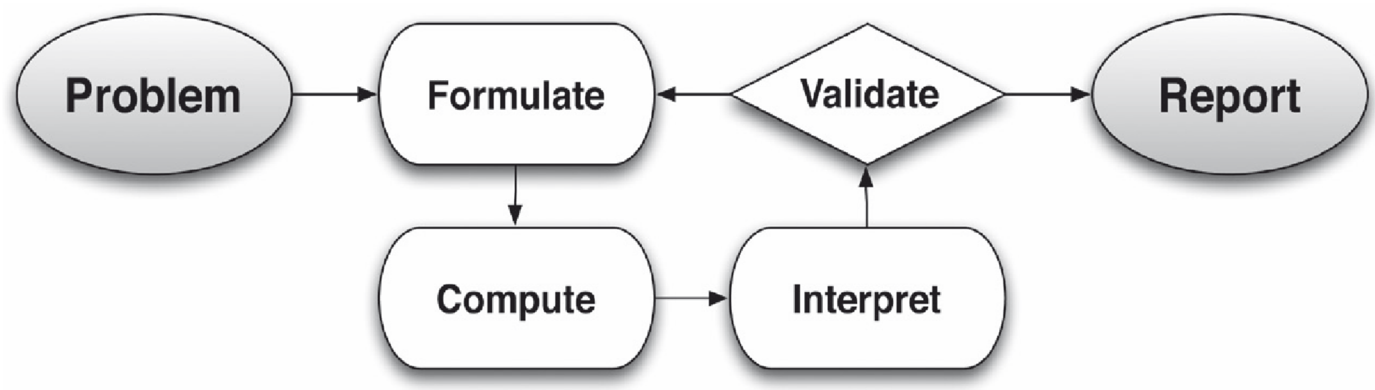
Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of three million people, and how it might be distributed.
- Planning a table tennis tournament for seven players at a club with four tables, where each player plays against each other.
- Designing the layout of the stalls in a school fair to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, for example, applied to turnaround of a plane at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on several factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from several types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO^2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

The table below shows an overview of the courses the standards may fall into. All the high school mathematics standards must fall within a three-year sequence that all students take. Major clusters are bolded.

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Note: star icon in a standard indicates where educators can integrate the Standards of Mathematical Practices, SMP 4, model with mathematics into their resources. The modeling and data analysis SMP bundle may also appear and indicate to include SMP 2, 4, and 5.

When including the model and data analysis bundle, students analyze complex, real-world scenarios and construct and use mathematical models to interpret and solve problems. When both the star and modeling and data analysis are indicated, educators should emphasize SMP 4 and the SMP bundle, model, and data analysis. When both are listed, educators will focus on the following modeling and data analysis characteristics:

- Apply mathematics to solve problems arising in everyday life, society, and the workplace.
- Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.
- State logical assumptions being used.
- Interpret results in the context of a situation.
- Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

Mathematics | High School—Conceptual Category: Number and Quantity N

Numbers and Number Systems. Students must repeatedly extend their conception of numbers from kindergarten to eighth grade. At first, “number” means “counting numbers”: 1, 2, 3... Soon after that, zero represents “none,” and the whole numbers are formed by counting numbers together with zero. The following extension is fractions. At first, fractions are barely numbers and tied firmly to pictorial representations. Yet by the time students understand the division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. In middle school, students augment fractions with negative fractions to form rational numbers. In Grade 8, students extend this system again, augmenting the rational numbers with the irrational numbers to create the real numbers. In high school, students encounter another extension of numbers when the imaginary numbers extend the real numbers to form the complex numbers.

With each number extension, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system — integers, rational numbers, real numbers, and complex numbers — the four operations stay the same in two important ways: They have commutative, associative, and distributive properties, and their new meanings are consistent with their earlier meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{\frac{1}{3}})^3$ should be $(5^{\frac{1}{3}})^3 = 5^1 = 5$ and that $(5^{\frac{1}{3}})$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can help students become better acquainted with these new number systems and their notation. They can generate data for numerical experiments, help understand the workings of matrix, vector, and complex number algebra, and experiment with non-integer exponents.

Quantities. In real-world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, per year per driver, or per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is essential for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also crucial for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Mathematics | High School—Conceptual Category: Algebra A

Expressions. An expression is a record of a computation with numbers, symbols representing numbers, arithmetic operations, exponentiation, and, at more advanced levels, evaluating a function. Conventions about using parentheses and the order of operations ensure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the calculation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analyzing its underlying structure. The study may suggest a different but equivalent way of writing the expression that exhibits various aspects of its meaning. For example, one can interpret $p + 0.05p$ as adding a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

The properties of operations and exponents, along with the conventions of algebraic notation, govern algebraic manipulations. At times, an expression results from applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the more straightforward expressions p and $0.05\, p$. Viewing an expression as the result of an operation on simpler expressions can sometimes clarify its underlying structure.

Students can use a spreadsheet or a computer algebra system (CAS) to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions. These values are the solutions to the equation. In contrast, mathematicians develop identities by rewriting an expression in an equivalent form, ensuring it holds for all values of the variables.

Equations yield a set of numbers as solutions in one variable, while in two variables, equations yield a set of ordered pairs of numbers, which one can plot in the coordinate plane. Two or more equations and inequalities form a system. A solution for such a system must satisfy every equation and inequality.

One can often solve an equation by successively deducing one or more simpler equations from it. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system but have a solution in a more extensive system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not actual numbers.

The same solution techniques used to solve equations can rearrange formulas. For example, the formula for the area of a trapezoid, $A = \left((\frac{b\,1+b_2}{2})h\right)$, can be solved for h using the same deductive process.

Reasoning about the properties of inequality can solve inequalities. Many, but not all, of the properties of equality, continue to hold for inequalities and can help solve them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions represent the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Mathematics | High School—Conceptual Category: Functions F

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the time supported. Because we continually make theories about dependencies between quantities in nature and society, functions are essential in constructing mathematical models.

In school mathematics, functions usually have numerical inputs and outputs, often defined by an algebraic expression. For example, the time it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = \frac{100}{v}$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph), by a verbal rule, as in, "I'll give you a state, you give me the capital city"; by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a helpful way of visualizing the relationship of the function models and manipulating a mathematical expression for a function can illuminate the function's properties.

Functions presented as expressions can model many critical phenomena. Two essential family functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a regular percentage rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can experiment with the properties of these functions and their graphs and build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations whose solutions we can visualize from the intersection of their graphs. Since functions describe relationships between quantities, people often use them in modeling. Sometimes, people define functions through a recursive process, which they can effectively display using a spreadsheet or other technology.

Mathematics | High School—Conceptual Category: Geometry G

Functions the attributes and relationships of geometric objects can be applied in diverse contexts. For instance, it can help interpret a schematic drawing of a building plan, estimate the amount of wood needed to frame a house's sloping roof, make computer graphics for a video game, or design a sewing pattern for the most efficient use of material in a fashion design project.

Although there are many geometry types, school mathematics is devoted primarily to plane Euclidean geometry, studied synthetically (without coordinates) and analytically (with coordinates). The parallel postulate most notably characterizes Euclidean geometry, stating that there is exactly one parallel line through a point, not on a given line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college, some students carefully create Euclidean and other geometries from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamentals are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and an object's symmetries offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For example, for triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, students learn this by drawing triangles from given conditions and noticing ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (Angle-Side-Angle (ASA), Side-Angle-Side (SAS), and Side-Side-Side, (SSS)) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to solve a triangle completely. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two sizes. The correspondence between numerical coordinates and geometric points enables the application of methods from algebra to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in the same way as computer algebra systems will enable them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points enables the application of methods from algebra to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation a tool for geometric understanding, modeling, and proof.

Mathematics | High School—Conceptual Category: Statistics and Probability S★

Decisions or predictions are often based on data — numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. For example, a business might use statistical analysis to decide whether to launch a new product based on sales data for comparable products.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. For instance, a company might collect data on customer purchases to understand buying trends. Quantitative data can be described in terms of key characteristics: shape, center, and spread measures. The shape of a data distribution might be described as symmetric, skewed, flat, or bell-shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean depend on the question to be investigated and the real-life actions to be taken.

Randomization has two critical uses in drawing statistical conclusions. First, collecting data from a random population sample makes it possible to draw valid conclusions about the total population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is unlikely to be due to chance alone. When critically reviewing the uses of statistics in public media and other reports, it's crucial to consider the study design, data collection methods, analyses employed, and the data summaries and conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. A probability model is like a list of all possible outcomes of a random event, like flipping a coin or rolling a dice, with each outcome having a specific chance of happening. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays a vital role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients quickly and to simulate many possible outcomes.

Connections to Functions and Modeling. If the data suggest a linear relationship, they can use functions to model the relationship with a regression line and express its strength and direction through a correlation coefficient.

High School Required Standards by Course (Algebra 1, Geometry, Algebra 2)

Conceptual Category	Domain	Algebra 1	Geometry	Algebra 2
Number and Quantity	The Real Number System	N-RN.B.3		**N-RN.A.1** **N-RN.A.2**
	Quantities ★	N-Q.A.1 N-Q.A.2 N-Q.A.3		N-Q.A.1 N-Q.A.2 N-Q.A.3
	The Complex Number System			N-CN.A.1 N-CN.A.2 N-CN.C.7
Algebra	Seeing Structure in Expressions	**A-SSE.A.1** **A-SSE.A.2** A-SSE.B.3		**A-SSE.A.1** **A-SSE.A.2** **A-SSE.B.3** **A-SSE.B.4**
	Arithmetic with Polynomials and Rational Expressions			**A-APR.A.1** **A-APR.B.2** **A-APR.B.3** A-APR.D.6
	Creating Equations ★	**A-CED.A.1** **A-CED.A.2** **A-CED.A.3** **A-CED.A.4**		A-CED.A.1 A-CED.A.2 A-CED.A.3 A-CED.A.4
	Reasoning with Equations and Inequalities	**A-REI.A.1** **A-REI.A.2** **A-REI.B.3** **A-REI.B.4** A-REI.C.5 A-REI.C.6 A-REI.C.7 **A-REI.D.10** **A-REI.D.11** **A-REI.D.12**		**A-REI.A.1** **A-REI.A.2** A-REI.B.4 A-REI.C.7 **A-REI.D.10** **A-REI.D.11** **A-REI.D.12**
Functions	Interpreting Functions	**F-IF.A.1** **F-IF.A.2** **F-IF.A.3** **F-IF.B.4** **F-IF.B.5** **F-IF.B.6** F-IF.C.7 F-IF.C.8		**F-IF.B.4** **F-IF.B.5** **F-IF.B.6** F-IF.C.7 F-IF.C.8
	Building Functions	F-BF.A.1 F-BF.A.2 F-BF.B.3		**F-BF.A.1** **F-BF.A.2** F-BF.B.3 F-BF.B.4

Conceptual Category	Domain	Algebra 1	Geometry	Algebra 2
	Linear, Quadratic, and Exponential Models ★	F-LE.A.1 F-LE.A.2 F-LE.A.3 F-LE.B.5		F-LE.A.2 F-LE.A.4 F-LE.B.5
	Trigonometric Functions			F-TF.A.1 F-TF.A.2 F-TF.A.3 F-TF.A.5
Geometry	Congruence		G-CO.A.1 G-CO.A.2 G-CO.A.3 G-CO.A.4 G-CO.A.5 **G-CO.B.6** **G-CO.B.7** **G-CO.B.8** **G-CO.C.9** **G-CO.C.10** **G-CO.C.11** G-CO.D.12 G-CO.D.13	
	Similarity, Right Triangles, and Trigonometry		**G-SRT.A.1** **G-SRT.A.2** **G-SRT.A.3** **G-SRT.B.4** **G-SRT.B.5** **G-SRT.C.6** **G-SRT.C.7** **G-SRT.C.8**	
	Circles		G-C.A.1 G-C.A.2 G-C.A.3 G-C.B.5	
	Expressing Geometric Properties with Equations		G-GPE.A.1 **G-GPE.B.4** **G-GPE.B.5** **G-GPE.B.6** **G-GPE.B.7**	G-GPE.A.2
	Geometric Measurement and Dimension		G-GMD.A.1 G-GMD.A.2 G-GMD.A.3 G-GMD.B.4	
	Modeling with Geometry		**G-MG.A.1** **G-MG.A.2** **G-MG.A.3**	

Conceptual Category	Domain	Algebra 1	Geometry	Algebra 2
Statistics and Probability ★	Interpreting Categorical and Quantitative Data ★	S-ID.A.1 S-ID.A.2 S-ID.A.3 S-ID.B.5 S-ID.B.6 **S-ID.C.7** **S-ID.C.8** **S-ID.C.9**		S-ID.A.4
	Making Inferences and Justifying Conclusions ★			S-IC.A.1 S-IC.A.2 **S-IC.B.3** **S-IC.B.4** **S-IC.B.5** **S-IC.B.6**
	Conditional Probability and the Rules of Probability ★		**S-CP.A.1** **S-CP.A.2** **S-CP.A.3** **S-CP.A.4** **S-CP.A.5** S-CP.B.6 S-CP.B.7	
	Using Probability to Make Decisions ★		S-MD.B.7	

Algebra 1

Narrative

Students in Algebra 1 formalize and extend the mathematics they learned in middle grades. They also apply mathematics to real-world problems, using newly discovered skills to solve more straightforward applications and using skills familiar from middle grades to complete more substantial modeling tasks.

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. In Algebra 1, students' study and explain the process of solving an equation and justify the process used in solving a system of equations. Students gain fluency in writing, interpreting, and translating various forms of linear equations and inequalities and use the equations and inequalities to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students become fluent with algebraic manipulation, including rearranging and collecting terms and factoring.

Both grade 8 and Algebra 1 students study different ways to solve systems of linear equations. In eighth grade, students are reminded of the significance of what a point represents on a line and explore what a point of intersection means using graphing and substitution. In Algebra 1, students explore the elimination strategy and more systems that may require manipulation of equations to solve algebraically.

In earlier grades, students defined, evaluated, and compared functions and used them to model relationships between quantities. In Algebra 1, students learn function notation and the concepts of domain and range. They interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations.

Building on and extending their understanding of integer exponents, students in Algebra 1 consider exponential functions. They compare linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities and find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Students extend the laws of exponents to rational exponents, connecting rational exponents to square and cube roots and applying this new understanding of numbers. They learn to see structure in quadratic and exponential expressions and create and solve equations, inequalities, and systems of equations involving quadratic expressions. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions and selecting from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. They identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to explore more specialized functions—absolute value, step, and piecewise-defined.

Building upon their prior experiences with data, students explore a more formal means of assessing how a model fits data. They use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of context to make judgments about the appropriateness of linear models. With linear models, students look at residuals to analyze the goodness of fit.

When a standard appears in both Algebra 1 and Algebra 2, the designation (A1 and A2) is indicated at the end of the standard.

Algebra 1 Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Clusters are listed below. The code, A1.N-RN denotes Algebra 1 **course**, numbers **conceptual category**, and the real number system **domain.**

The Real Number System (N-RN)

- Use properties of rational and irrational numbers.

Quantities (N-Q)

- Reason quantitatively and use units to solve problems.

Seeing Structure in Expressions (A-SSE)

- **Interpret the structure of expressions.**
- Write expressions and equations in equivalent forms to solve problems.

Creating Equations ★ (A-CED)

- **Create equations that describe numbers or relationships.**

Reasoning with Equations and Inequalities (A-REI)

- **Understand solving equations as a process of reasoning and explain the reasoning.**
- **Solve equations and inequalities in one variable.**
- Solve systems of equations.
- **Represent and solve equations and inequalities graphically.**

Interpreting Functions (F-IF)

- **Understand the concept of a function and use function notation.**
- **Interpret functions that arise in applications in terms of the context.**
- **Analyze functions using different representations.**

Building Functions (F-BF)

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Linear, Quadratic and Exponential Models (F-LE)

- Construct and compare linear, quadratic, and exponential models and solve.
- Interpret expressions for functions in terms of the situation they model.

Interpreting Categorical and Quantitative Data ★(S-ID)

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- **Interpret linear models.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Conceptual Category: A1.N Number and Quantity

Domain: A1.N-RN The Real Number System

A1.N-RN.B Additional Cluster: Use properties of rational and irrational numbers.

Standard	Rigor	SMP Bundle
A1.N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	Conceptual	Communicating Reasoning

Domain: A1.N-Q Quantities ★

A1.N-Q.A Supporting Cluster: Reason quantitatively and use units to solve problems.

Standard	Rigor	SMP Bundle
A1.N-Q.A.1 Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (A1 and A2)	Conceptual Application	Problem Solving Modeling & Data Analysis
A1.N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.	Conceptual	Problem Solving Modeling & Data Analysis
A1.N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	Conceptual Application	Problem Solving Modeling & Data Analysis

Conceptual Category: A1.A Algebra

Domain: A1.A-SSE Seeing Structure in Expressions and Equations

A-SSE.A Major Cluster: Interpret the structure of expressions.

Standard	Rigor	SMP Bundle
A1.A-SSE.A.1.a Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. ★	Conceptual Application	Problem Solving
A1.A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* (x^2+y^2).	Conceptual Procedural	Problem Solving Communicating Reasoning

A-SSE.A Supporting Cluster: Write expressions and equations in equivalent forms to solve problems.

Standard	Rigor	SMP Bundle
A1.A-SSE.B.3 Choose and produce an equivalent form of an expression or equation to reveal and explain properties of the quantity represented by the expression. ★ (A1 and A2) a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic equation to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 3^x *can be rewritten as* $(1+2)^x$ *to reveal the growth rate is* 200%.	Conceptual Procedural Application	Problem Solving Communicating Reasoning

Domain: A1.A-CED Creating Equations ★

A1.A-CED.A Major Cluster: Create equations that describe numbers or relationships.

Standard	Rigor	SMP Bundle
A1.A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	Conceptual Application	Problem Solving Modeling & Data Analysis
A1.A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	Conceptual Application	Problem Solving Modeling & Data Analysis
A1.A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	Conceptual Application	Problem Solving Modeling & Data Analysis
A1.A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.	Procedural	Problem Solving Modeling & Data Analysis

Domain: A1.A-REI Reasoning with Equations and Inequalities

A1.A-REI.A Major Cluster: Understand solving equations as a process of reasoning and explain the reasoning.

Standard	Rigor	SMP Bundle
A1.A-REI.A.1 Explain each step-in solving equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A1 and A2)	Conceptual	Communicating Reasoning Modeling & Data Analysis
A1.A-REI.A.2 Solve rational and radical equations in one variable and give examples showing how extraneous solutions may arise. (A1 and A2)	Conceptual Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis

A1.A-REI.B Major Cluster: Solve equations and inequalities in one variable.

Standard	Rigor	SMP Bundle
A1.A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Procedural	Problem Solving Modeling & Data Analysis
A1.A-REI.B.4 Solve quadratic equations in one variable. (A1 and A2) a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. b. Solve quadratic equations with real solutions using any method.	Procedural	Problem Solving Modeling & Data Analysis

A-REI.C Additional Cluster: Solve systems of equations.

Standard	Rigor	SMP Bundle
A1.A-REI.C.5 Explain how the strategy of elimination results in finding solution(s) to a system of equations.	Conceptual Procedural	Problem Solving Communicating Reasoning
A1.A-REI.C.6 Solve systems of linear equations exactly and approximately. *For example, with graphs, focusing on pairs of linear equations in two variables.*	Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis
A1.A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (A1 and A2)	Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis

A1.A-REI.D Major Cluster: Represent and solve equations and inequalities graphically.

Standard	Rigor	SMP Bundle
A1.A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A1 and A2)	Procedural	Problem Solving Communicating Reasoning
A1.A-REI.D.11 Explain why the solution(s) of a system of equations are the point(s) of intersection(s) on a coordinate plane. Find the solutions approximately, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, absolute value functions. ★ (A1 and A2)	Conceptual Procedural	Problem Solving Communicating Reasoning
A1.A-REI.D.12 Graph and interpret (with the use of technology) the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (A1 and A2)	Procedural Application	Problem Solving

Conceptual Category: A1.F Functions

Domain: A1.F-IF Interpreting Functions

A1.F-IF.A Major Cluster: Understand the concept of a function and use function notation.

Standard	Rigor	SMP Bundle
A1.F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input f. The graph of f is the graph of the equation $y = f(x)$.	Conceptual	Problem Solving Communicating Reasoning
F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	Procedural	Problem Solving
A1.F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$*for* $n \geq 1$.	Conceptual	Problem Solving

A1.F-IF.B Major Cluster: Interpret functions that arise in applications in terms of the context.

Standard	Rigor	SMP Bundle
A1.F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features may include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximum and minimum; and symmetries. ★	Conceptual Application	Problem Solving Modeling & Data Analysis
A1.F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A1.F-IF.B.6 Calculate and interpret the average rate of change of a nonlinear function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A1 and A2)	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

A1.F-IF.C Major Cluster: Analyze functions using different representations.

Standard	Rigor	SMP Bundle
A1.F-IF.C.7 Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases. ★ a. Graph linear and quadratic functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases, including intercepts, maxima, and minima if they exist. ★ (A1 and A2)	Procedural	Problem Solving Modeling & Data Analysis
A1.F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. *For example, use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.* (A1 and A2)	Procedural Application	Problem Solving Modeling & Data Analysis

Domain: A1.F-BF Building Functions

A1.F-BF.A Supporting Cluster: Build a function that models a relationship between two quantities.

Standard	Rigor	SMP Bundle
A1.F-BF.A.1a Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. ★	Conceptual	Problem Solving Modeling & Data Analysis
A1.F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Interpret arithmetic sequences as linear functions and geometric sequences as exponential functions. ★ (A1 and A2)	Procedural Application	Problem Solving Modeling & Data Analysis

A1.F-BF.B Additional Cluster: Build new functions from existing functions.

Standard	Rigor	SMP Bundle
A1.F-BF.B.3 Identify the effect on linear and quadratic graphs of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: A1.F-LE Linear, Quadratic, and Exponential Models ★

A1.F-LE.A Supporting Cluster: Construct and compare linear, quadratic, and exponential models and solve problems.

Standard	Rigor	SMP Bundle
A1.F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	Conceptual Procedural Application	Modeling & Data Analysis
A1.F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (A1 and A2)	Procedural	Modeling & Data Analysis
A1.F-LE.A.3 Use graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	Procedural	Modeling & Data Analysis

A1.F-LE.B Supporting Cluster: Interpret expressions for functions in terms of the situation they model.

Standard	Rigor	SMP Bundle
A1.F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context. (A1 and A2)	Conceptual Application	Modeling & Data Analysis

Conceptual Category: A1.S Statistics and Probability ★

Domain: A1.S-ID Interpreting Categorical and Quantitative Data ★

A1.S-ID.A Additional Cluster: Summarize, represent, and interpret data on a single count or measurement variable.★

Standard	Rigor	SMP Bundle
A1.S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots) in a modeling context. ★	Procedural	Modeling & Data Analysis
A1.S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ★	Conceptual	Modeling & Data Analysis
A1.S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★	Conceptual Application	Modeling & Data Analysis

A1.S-ID.B Supporting Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.★

Standard	Rigor	SMP Bundle
A1.S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★	Conceptual	Modeling & Data Analysis
A1.S-ID.B.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related. ★ a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	Conceptual Procedural Application	Modeling & Data Analysis

A1.S-ID.C Major Cluster: Interpret linear models.★

Standard	Rigor	SMP Bundle
A1.S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★	Conceptual	Problem Solving
A1.S-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. ★	Conceptual Procedural	Problem Solving
A1.S-ID.C.9 Distinguish between correlation and causation. ★	Conceptual	Problem Solving

Geometry

Narrative

Geometry is not just about abstract shapes and theorems. It's a practical tool that helps us understand and navigate the world. In this course, students will formalize and extend the mathematics they learned in the middle grades. They will explore more complex geometric situations and deepen their explanations of geometric relationships by presenting and hearing formal mathematical arguments. They will also solve real-world problems by combining geometric methods, algebra techniques, and numerical calculations.

High school students have already built a solid foundation in geometry. They have experience drawing triangles based on given measurements and performing rigid motions, including translations, reflections, and rotations. Students have used these to form ideas about what it means for two objects to be congruent. This course will take their understanding to the next level. Students will establish triangle congruence criteria based on analyses of rigid motions and formal constructions. Students will use triangle congruence as a familiar foundation for learning formal proof, proving theorems using a variety of formats, including deductive and inductive reasoning and proof by contradiction. Students will apply reasoning to complete geometric constructions and explain why they work. Students will use geometric relationships to solve problems about figures.

Students extend their previous experience with dilations to gain a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to solve problems in right triangle trigonometry.

Students extend their experience with three-dimensional objects to include informally explaining circumference, area, and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Building on their work with the Pythagorean Theorem in middle grades to find distances, students use the rectangular coordinate system to verify geometric relationships, including slopes of parallel and perpendicular lines, which relates to work done in Algebra 1. Students continue their study of quadratics by solving applied problems involving geometric measurement leading to quadratic equations and by connecting the geometric and algebraic definitions of the parabola.

In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane and apply techniques for solving quadratic equations to determine intersections between lines and circles or parabolas and between two circles.

Get ready to dive deeper into the fascinating world of probability! Building on the concept, students started exploring in the middle grades, we will use the language of sets to expand their ability to work with probability for compound events. We will pay special attention to mutually exclusive events, independent events, and conditional probability. And most importantly, we will use probability to make informed decisions. It's not just about numbers and calculations; it's about understanding the world around us and making intelligent choices

Geometry Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Clusters are listed below. The code, G.G-CO denotes geometry course, geometry conceptual category, and congruence domain.

Congruence (G-CO)

- Experiment with transformations in the plane.
- **Understand congruence in terms of rigid motions.**
- **Prove geometric theorems.**
- Make geometric constructions.

Similarity, Right Triangles, and Trigonometry (G-SRT)

- **Understand similarity in terms of similarity transformations.**
- **Prove theorems using similarity.**
- **Define trigonometric ratios and solve problems involving right triangles.**

Circles (G-C)

- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles.

Expressing Geometric Properties with Equations (G-GPE)

- Translate between the geometric description and the equation for a conic section.
- **Use coordinates to prove simple geometric theorems algebraically.**

Geometric Measurement and Dimension (G-GMD)

- Explain volume formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects.

Modeling with Geometry (G-MG)

- **Apply geometric concepts in modeling situations.**

Conditional Probability and the Rules of Probability ★ (S-CP)

- **Use independence and conditional probability to interpret data.**
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Use Probability to Make Decisions ★ (S-CP)

- Use probability to evaluate outcomes of decisions

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Conceptual Category: G.G Geometry

Domain: G.G-CO Congruence

G.G-CO.A Supporting Cluster: Experiment with transformations in the plane.

Standard	Rigor	SMP Bundle
G.G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	Conceptual	Communicating Reasoning
G.G-CO.A.2 Represent transformations in the plane using geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not. *For example, translation versus horizontal stretch).*	Conceptual	Communicating Reasoning
G.G-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	Conceptual	Communicating Reasoning
G.G-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	Conceptual	Communicating Reasoning
G.G-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another. *For example, using graph paper, tracing paper, or geometry software.*	Conceptual	Communicating Reasoning

G.G-CO.B Major Cluster: Understand congruence in terms of rigid motions.

Standard	Rigor	SMP Bundle
G.G-CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	Conceptual	Communicating Reasoning
G.G-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	Conceptual	Communicating Reasoning
G.G-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	Conceptual	Communicating Reasoning

G.G-CO.C Major Cluster: Prove geometric theorems.

Standard	Rigor	SMP Bundle
G.G-CO.C.9 Prove theorems about lines and angles. Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	Conceptual	Communicating Reasoning
G.G-CO.C.10 Prove theorems about parallelograms. Theorems include opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	Conceptual	Communicating Reasoning
G.G-CO.C.11 Prove theorems about parallelograms. Theorems include opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	Conceptual	Communicating Reasoning

G.G-CO.D Major Cluster: Make geometric constructions.

Standard	Rigor	SMP Bundle
G.G-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	Conceptual Procedural	Communicating Reasoning
G.G-CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	Conceptual	Communicating Reasoning

Domain: G.G-SRT Similarity, Right Triangles, and Trigonometry

G.G-SRT.A Major Cluster: Understand similarity in terms of similarity transformations.

Standard	Rigor	SMP Bundle
G.G-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	Conceptual	Communicating Reasoning
G.G-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	Conceptual	Communicating Reasoning
G.G-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	Conceptual	Communicating Reasoning

G.G-SRT.B Major Cluster: Prove and apply theorems involving similarity.

Standard	Rigor	SMP Bundle
G.G-SRT.B.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	Conceptual	Communicating Reasoning
G.G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	Conceptual	Communicating Reasoning

G.G-SRT.C Major Cluster: Define trigonometric ratios and solve problems involving right triangles.

Standard	Rigor	SMP Bundle
G.G-SRT.C.6 Define trigonometric ratios using similar triangle ratios and the ratios of corresponding side lengths.	Conceptual Procedural	Problem Solving Communicating Reasoning
G.G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.	Conceptual Procedural	Problem Solving Communicating Reasoning

Standard	Rigor	SMP Bundle
G.G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems, including special right triangles. ★	Conceptual Procedural	Problem Solving Communicating Reasoning

Domain: G.G-C Circles

G.G-C.A Additional Cluster: Understand and apply theorems about circles.

Standard	Rigor	SMP Bundle
G.G-C.A.1 Prove that all circles are similar.	Conceptual	Communicating Reasoning
G.G-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	Conceptual	Communicating Reasoning
G.G-C.A.3 Construct the inscribed and circumscribed circles of a triangle with technology, and investigate properties of a quadrilateral inscribed in a circle.	Conceptual	Communicating Reasoning

G.G-C.B Additional Cluster: Find arc lengths and areas of sectors of circles.

Standard	Rigor	SMP Bundle
G.G-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	Conceptual	Communicating Reasoning

Domain: G.G-GPE Expressing Geometric Properties with Equations

G.G-GPE.A Additional Cluster: Translate between the geometric description and the equation for a conic section.

Standard	Rigor	SMP Bundle
G.G-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	Conceptual	Communicating Reasoning

G.G-GPE.B Major Cluster: Use coordinates to prove simple geometric theorems algebraically.

Standard	Rigor	SMP Bundle
G.G-GPE.B.4 Use coordinate geometry to prove simple geometric theorems. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle.*	Conceptual	Communicating Reasoning
G.G-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. *For example, find the equation of a line parallel or perpendicular to a given line that passes through a given point.*	Conceptual	Communicating Reasoning
G.G-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Procedural	Problem Solving
G.G-GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. *For example, use the distance formula to calculate the distance between the two points.* ★	Conceptual	Problem Solving

Domain: G.G-GMD Geometric Measurement and Dimension

G.G-GMD.A Additional Cluster: Explain volume formulas and use them to solve problems.

Standard	Rigor	SMP Bundle
G.G-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	Conceptual	Communicating Reasoning Modeling & Data Analysis
G.G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★	Procedural Application	Problem Solving

G.G-GMD.B Additional Cluster: Visualize relationships between two-dimensional and three-dimensional objects.

Standard	Rigor	SMP Bundle
G.G-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	Conceptual	Problem Solving Modeling & Data Analysis

Domain: G.G-MG Modeling with Geometry

G.G-MG.A Major Cluster: Apply geometric concepts with modeling situations.

Standard	Rigor	SMP Bundle
G.G-MG.A.1 Use geometric shapes, their measures, and their properties to describe and explain objects. For example, modeling a tree trunk as a cylinder. ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
G.G-MG.A.2 Apply concepts of density based on area and volume in modeling situations. *For example, persons per square mile, BTUs per cubic foot.* ★	Conceptual Application	Problem Solving Modeling & Data Analysis
G.G-MG.A.3 Apply geometric methods to solve design problems. *For example, designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.* ★	Procedural Application	Problem Solving Modeling & Data Analysis

Conceptual Category: G.S-CP Statistics and Probability

Domain: G.S-CP Conditional Probability and the Rules of Probability★

G.S-CP.A Additional Cluster: Use independence and conditional probability to interpret data.★

Standard	Rigor	SMP Bundle
G.S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or” “and” “not”). ★	Conceptual Procedural	Problem Solving
G.S-CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent. ★	Conceptual Procedural	Problem Solving
G.S-CP.A.3 Understand the conditional probability of A given B as $\frac{P(A \text{ and } B)}{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★	Conceptual Procedural	Problem Solving
G.S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* ★	Procedural Application	Problem Solving
G.S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* ★	Conceptual Procedural Application	Problem Solving

G.S-CP.B Additional Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model. ★

Standard	Rigor	SMP Bundle
G.S-CP.B.6 Find the conditional probability of A given B as the fraction of $B's$ outcomes that also belong to A and interpret the answer in terms of the model. ★	Procedural Application	Problem Solving
G.S-CP.B.7 Apply the Addition Rule, $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$, and interpret the answer in terms of the model.★	Procedural Application	Problem Solving

Domain: G.S-MD Using Probability to Make Decisions ★

G.S-MD.B Supporting Cluster: Use probability to evaluate outcomes of decisions. ★

Standard	Rigor	SMP Bundle
G.S-MD.B.7 Analyze decisions and strategies using probability concepts. *For example, product testing, medical testing, pulling a hockey goalie at the end of a game.* ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Algebra 2

Narrative

Students in Algebra 2 formalize and extend the mathematics they learned in Algebra 1 and apply these mathematical concepts to real-world problems. This practical application of their newly discovered skills helps them solve more straightforward applications. It equips them to tackle more substantial modeling tasks, making their learning experience more relevant and impactful.

Algebra 2 students will extend their work with quadratic and exponential functions and advance their understanding of function families to include polynomial, rational, and logarithmic functions. This progression in their knowledge and skills will continue to expand and hone their abilities to model situations and solve equations.

In Algebra 2, students discover that the arithmetic of polynomial expressions follows the same rules as the arithmetic of integers. Students can draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Connections are available between the multiplication of polynomials and the multiplication of multi-digit integers and between the division of polynomials and the long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations.

One of the critical aspects of this course is the modeling process. Students synthesize and generalize what they have learned about a variety of function families, extending their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. This process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions is at the heart of this course, emphasizing its significance and complexity.

Students see how the visual displays and summary statistics they learned in earlier grades relate to different data types and probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulation—and recognize the role of randomness and careful design in shaping the conclusions drawn.

Algebra 2 Overview

Major clusters are **bolded**; these represent the skills and concepts that should represent 65–85% of instructional time.

Clusters are listed below. The code, A2.N-RN denotes Algebra 2 **course**, numbers **conceptual category**, and real number **domain.**

The Real Number System (N-RN)

- **Extend the properties of exponents to rational exponents.**

Quantities ★ (N-Q)

- Reason quantitatively and use units to solve problems.

The Complex Number System (N-CN)

- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities and equations.

Seeing Structure in Expressions (A-SSE)

- **Interpret the structure of expressions.**
- **Write expressions in equivalent forms to solve problems.**

Arithmetic with Polynomials and Rational Expressions (A-APR)

- **Perform arithmetic operations on polynomials.**
- **Understand the relationship between zeros and factors of polynomials.**
- Rewrite rational expressions.

Reasoning with Equations and Inequalities (A-REI)

- **Understand solving equations as a process of reasoning and explain the reasoning.**
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- **Represent and solve equations and inequalities graphically.**

Interpreting Functions (F-IF)

- **Interpret functions that arise in applications in terms of the context.**
- Analyze functions using different representations.

Building Functions (F-BF)

- **Build a function that models a relationship between two quantities.**
- Build new functions from existing functions.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Linear, Quadratic, and Exponential Models (F-LE)

- Construct and compare linear, quadratic, and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

Trigonometric Functions (F-TF)

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.

Interpreting Categorical and Quantitative Data ★ (S-ID)

- Summarize, represent, and interpret data on a single count or measurement variable.

Making Inferences and Justifying Conclusions ★ (S-IC)

- Understand and evaluate random processes underlying statistical experiments.
- **Make inferences and justify conclusions from sample surveys, experiments and observational studies.**

Conceptual Category: A2.N Number and Quantity

Domain: A2.N-RN The Real Number System

A2.N-RN.A Major Cluster: Extend the properties of exponents to rational exponents.

Standard	Rigor	SMP Bundle
A2.N-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{\frac{1}{3}}$ *to be the cube root of 5 because we want* $(5^{\frac{1}{3}})^3 = 5^{\left(\frac{1}{3}\right)^3}$ *to hold, so* $5^{\left(\frac{1}{3}\right)^3}$ *must equal 5.*	Conceptual	Communicating Reasoning
A2.N-RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Conceptual Application	Communicating Reasoning

Domain: A2.N-Q Quantities ★

A2.N-Q.A Supporting Cluster: Reason quantitatively and use units to solve problems.

Standard	Rigor	SMP Bundle
A2.N-Q.A.1 Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (A1 and A2)	Conceptual Application	Problem Solving Modeling & Data Analysis
A2.N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.	Conceptual	Problem Solving Modeling & Data Analysis
A2.N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	Conceptual Application	Problem Solving Modeling & Data Analysis

Domain:A2.N-CN The Complex Number System

A2.N-CN.A Additional Cluster: Perform arithmetic operations with complex numbers.

Standard	Rigor	SMP Bundle
A2.N-CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	Conceptual	Communicating Reasoning
A2.N-CN.A.2 Use the relation $i^2 = -1$and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Procedural	Problem Solving

A2.N-CN.B Additional Cluster: Use complex numbers in polynomial identities and equations.

Standard	Rigor	SMP Bundle
A2.N-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.	Procedural	Problem Solving

Conceptual Category: A2.A Algebra

Domain: A2.A-SSE Seeing Structure in Expressions and Equations

A2.A-SSE.A Major Cluster: Interpret the structure of expressions.

Standard	Rigor	SMP Bundle
2.A-SSE.A.1b Interpret expressions that represent a quantity in terms of its context. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* ★	Procedural Application	Problem Solving
A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	Conceptual Procedural	Problem Solving Communicating Reasoning

A2.A-SSE.A Major Cluster: Write expressions and equations in equivalent forms to solve problems.

Standard	Rigor	SMP Bundle
A2.A-SSE.B.3 Choose and produce an equivalent form of an expression or equation to reveal and explain properties of the quantity represented by the expression. (A1 and A2) a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic equation to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 3^x *can be rewritten as* $(1+2)^x$ *to reveal the growth rate is 200%.* ★	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis
A2.A-SSE.B.4 Choose and produce an equivalent form of an expression or equation to reveal and explain properties of the quantity represented by the expression. ★	Procedural Application	Problem Solving Modeling & Data Analysis

Domain: A2.A-APR Arithmetic with Polynomials and Rational Expressions

A2.A-APR.A Major Cluster: Perform arithmetic operations on polynomials.

Standard	Rigor	SMP Bundle
A2.A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Conceptual	Communicating Reasoning

A2.A-APR.B Major Cluster: Understand the relationship between zeros and factors of polynomials.

Standard	Rigor	SMP Bundle
A2.A-APR.B.2 Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	Conceptual Procedural	Communicating Reasoning
A2.A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.	Conceptual Procedural	Communicating Reasoning

A2.A-APR.D Supporting Cluster: Rewrite rational expressions.

Standard	Rigor	SMP Bundle
A-APR.D.6 Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x)$ + $\frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*). For example, in the same way one may view $\frac{11}{7}$ as $\frac{(7+4)}{7} = 1 + \frac{4}{7}$, one can view $\frac{(x+7)}{(x+3)}$ as $\frac{((x+3)+4)}{(x+3)} = 1 + \frac{4}{x+3}$	Conceptual Procedural	Communicating Reasoning

Domain: A2.A-CED Creating Equations ★

A2.A-CED.A Supporting Cluster: Create equations that describe numbers or relationships.

Standard	Rigor	SMP Bundle
A2.A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. (A1 and A2)	Conceptual Application	Problem Solving Modeling & Data Analysis
A2.A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A1 and A2)	Conceptual Application	Problem Solving Modeling & Data Analysis
A2.A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* (A1 and A2)	Conceptual Application	Problem Solving Modeling & Data Analysis
A2.A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange the formula for the area of a trapezoid, A =(b1 +b2)/2*h for the length of one of the bases.* (A1 and A2)	Procedural	Problem Solving Modeling & Data Analysis

Domain: A2.A-REI Reasoning with Equations and Inequalities

A2.A-REI.A Major Cluster: Understand solving equations as a process of reasoning and explain the reasoning.

Standard	Rigor	SMP Bundle
A2.A-REI.A.1 Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A1 and A2)	Conceptual	Communicating Reasoning Modeling & Data Analysis
A2.A-REI.A.2 Solve rational and radical equations in one variable and give examples showing how extraneous solutions may arise. (A1 and A2)	Conceptual Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis

A-REI.B Supporting Cluster: Solve equations and inequalities in one variable.

Standard	Rigor	SMP Bundle
A2.A-REI.B.4 Solve quadratic equations in one variable. (A1 and A2) a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. b. Solve quadratic equations with real solutions using any method.	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

A2.A-REI.C Additional Cluster: Solve systems of equations.

Standard	Rigor	SMP Bundle
A2.A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (A1 and A2)	Procedural	Problem Solving Communicating Reasoning Modeling & Data Analysis

A-REI.D Major Cluster: Represent and solve equations and inequalities graphically.

Standard	Rigor	SMP Bundle
A2.A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A1 and A2)	Procedural	Problem Solving Communicating Reasoning
A2.A-REI.D.11 Explain why the solution(s) of a system of equations are the point(s) of intersection(s) on a coordinate plane. Find the solutions approximately. *For example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$) and/or $g(x)$ are quadratic, exponential, rational, absolute value functions, polynomial, exponential, and logarithmic functions.* ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning
A2.A-REI.D.12 Graph and interpret (with the use of technology) the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (A1 and A2)	Procedural Application	Problem Solving

Conceptual Category: A2.F Functions

Domain: A2.F-IF Interpreting Functions

A2.F-IF.B Major Cluster: Interpret functions that arise in applications in terms of the context.

Standard	Rigor	SMP Bundle
A2.F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features may include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximum and minimum; and symmetries. ★	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis
A2.F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F*or example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.F-IF.B.6 Calculate and interpret the average rate of change of a nonlinear function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A1 and A2)	Conceptual Procedural Application	Problem Solving Modeling & Data Analysis

A2.F-IF.C Supporting Cluster: Analyze functions using different representations.

Standard	Rigor	SMP Bundle
A2.F-IF.C.7 Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases. ★ (A1 and A2) a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. *For example, rewrite rational expressions to show the vertical transformation.*	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: F-BF Building Functions

A2.F-BF.A Major Cluster: Build a function that models a relationship between two quantities.

Standard	Rigor	SMP Bundle
A2.F-BF.A.1b Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.F-BF.A.2 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. *For example, rewrite rational functions to show the vertical transformation from the parent function.* (A1 and A2)	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

A2.F-BF.B Additional Cluster: Build new functions from existing functions.

Standard	Rigor	SMP Bundle
A2.F-BF.B.3 Identify the effect on linear and quadratic graphs of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A1 and A2)	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.F-BF.B.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or,* $f(x) = \frac{(x+1)}{(x-1)}$ *for* $x \neq 1$.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: F-LE Linear, Quadratic, and Exponential Models ★

A2.F-LE.A Supporting Cluster: Construct and compare linear, quadratic, and exponential models and solve problems.

Standard	Rigor	SMP Bundle
A2.F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (A1 and A2)	Procedural	Modeling & Data Analysis
A2.F-LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where ab c, and d are numbers and the base a is 2, 10, or e, evaluate the logarithm using technology.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

A2.F-LE.B Supporting Cluster: Interpret expressions for functions in terms of the situation they model.

Standard	Rigor	SMP Bundle
A2.F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context. (A1 and A2)	Conceptual Application	Modeling & Data Analysis

Domain: A2F-TF Trigonometric Functions

A2F-TF.A Additional Cluster: Extend the domain of trigonometric functions using the unit circle.

Standard	Rigor	SMP Bundle
A2.F-TF.A.1 Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.F-TF.A.3 Use special triangles to determine geometrically the values of sine, cosine for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine and cosine for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. Note: Does not include tangent.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.F-TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	Conceptual Procedural	Problem Solving Communicating Reasoning

A2F-TF.B Additional Cluster: Model periodic phenomena with trigonometric functions.

Standard	Rigor	SMP Bundle
A2.F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Conceptual Category: A2.S Statistics and Probability ★

Domain: A2.S-ID Interpreting Categorical and Quantitative Data ★

S-ID.A Additional Cluster: Summarize, represent, and interpret data on a single count or measurement variable. ★

Standard	Rigor	SMP Bundle
A2.S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: A2.S-IC Making Inferences and Justifying Conclusions ★

A2.S-IC.A Supporting Cluster: Understand and evaluate random processes underlying statistical experiments.★

Standard	Rigor	SMP Bundle
A2.S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.S-IC.A.2 Decide if a specified model is consistent with results from a given data-generating process. *For example, using simulation or a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

S-IC.B Major Cluster: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.★

Standard	Rigor	SMP Bundle
A2.S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.S-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error using simulation models for random sampling. ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.S-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis
A2.S-IC.B.6 Evaluate reports based on data. ★	Conceptual Procedural Application	Communicating Reasoning Modeling & Data Analysis

Mathematics Standards for High School Elective Courses (Statistics, Precalculus, Calculus)

Statistics and Probability

Narrative

Statistics and probability are both a conceptual category and a course. All of the required standards contained in the course are also included in the previous high school courses in this document. Similar to how precalculus and calculus courses develop the conceptual categories of algebraic and function concepts, the conceptual category of statistics and probability create a course. There are two distinctions beyond this for the Statistics and Probability course. The course includes an Iowa standard for students to conduct statistical investigations, both observational studies and statistical experiments. The conceptual category of modeling is emphasized throughout this course.

Decisions or predictions, crucial in various fields, are often based on data - numbers in context significantly affect our daily lives. However, the data doesn't always send a clear message due to variability. Statistics provides tools to describe this variability in data and make informed decisions that consider it. By understanding these tools, students can feel empowered to navigate the complexities of data analysis and make confident decisions.

Data, the foundation of statistical analysis, are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Key characteristics of quantitative data include shape, center, and spread measures. However, it's about more than just knowing these measures. Understanding the context and purpose of one’s analysis is crucial in deciding which statistics to compare, which plots to use, and what the results might mean in real-life actions.

Randomization, a critical statistical concept, serves two essential purposes: enabling valid conclusions about a total population by collecting data from a random sample and facilitating fair comparisons of treatment effectiveness. These theoretical concepts have direct real-world applications, enhancing the robustness and reliability of statistical conclusions. Statistical significance is determined by randomness, indicating outcomes unlikely due to chance alone. When critically reviewing statistics in media and reports, it's crucial to consider study design, data collection methods, analyses employed, data summaries, and conclusions drawn, as these conditions are essential for reaching accurate conclusions.

A probability model mathematically describes random processes by assigning probabilities to each possible outcome listed or described in the sample space. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to form events; students can compute event probabilities by applying the Addition and Multiplication Rules.

Interpreting these probabilities relies on understanding independence and conditional probability, which one can approach by analyzing two-way tables.

Technology plays a vital role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients quickly and to simulate many possible outcomes. This technological support ensures students are not alone in their data analysis journey, providing the tools students need to make informed decisions.

Connections to Functions and Modeling. If the data suggest a linear relationship, one can model it with a regression line using functions to describe data and express its strength and direction through a correlation coefficient.

Note: Cluster level emphasis (for example, Major, Supporting, and Additional designations) for this course is intentionally not included because the content is beyond the college and career readiness level for all students.

Statistics and Probability Overview

Clusters are listed below. The code, ST.S-ID denotes statistics and probability course, statistics **conceptual category**, interpreting categorical and quantitative data **domain**.

Interpreting Categorical and Quantitative Data ★ (S-ID)

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

Making Inferences and Justifying Conclusions ★ (S-IC)

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Conditional Probability and Rules of Probability ★ (S-CP)

- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Using Probability to Make Decisions ★ (S-MD)

- Calculate expected values and use them to solve problems.
- Use probability to evaluate outcomes of decisions.

Conditional Probability and Rules of Probability ★ (S-CP)

- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Note: Nearly every standard in this course has a star because the star symbol appears throughout the entire conceptual category in the Algebra 1, Geometry and Algebra 2 bundles.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Conceptual Category: ST.S Statistics and Probability ★

Domain: ST.S-ID Interpreting Categorical and Quantitative Data★

ST.S-ID.A Cluster: Summarize, represent, and interpret data on a single count or measurement variable. ★

Standard	Rigor	SMP Bundle
S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).★	Procedural	Modeling & Data Analysis
S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★	Conceptual Procedural Application	Modeling & Data Analysis
S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★	Conceptual Procedural Application	Modeling & Data Analysis
S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★	Conceptual Procedural Application	Modeling & Data Analysis

ST.S-ID.B Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.★

Standard	Rigor	SMP Bundle
ST.S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★	Conceptual Procedural Application	Modeling & Data Analysis
ST.S-ID.B.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related. ★ a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	Conceptual Procedural Application	Modeling & Data Analysis

ST.S-ID.C Cluster: Interpret linear models.★

Standard	Rigor	SMP Bundle
ST.S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★	Conceptual Procedural Application	Problem Solving
ST.S-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. ★	Conceptual Procedural Application	Problem Solving
ST.S-ID.C.9 Distinguish between correlation and causation. ★	Conceptual Procedural Application	Problem Solving

Domain: STS-IC Making Inferences and Justifying Conclusions ★

ST.S-IC.A Cluster: Understand and evaluate random processes underlying statistical experiments.★

Standard	Rigor	SMP Bundle
ST.S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★	Conceptual Procedural Application	Modeling & Data Analysis
ST.S-IC.A.2 Decide if a specified model is consistent with results from a given data-generating process. *For example, using simulation or a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★	Conceptual Procedural Application	Modeling & Data Analysis

ST.S-IC.B Cluster: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Standard	Rigor	SMP Bundle
ST.S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★	Conceptual Procedural Application	Modeling & Data Analysis
ST.S-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error using simulation models for random sampling. ★	Conceptual Procedural Application	Modeling & Data Analysis
ST.S-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★	Conceptual Procedural Application	Modeling & Data Analysis
ST.S-IC.B.6 Evaluate reports based on data. ★ *For example, a magazine poll reported on the status of American women. One of the statements in the poll was "It is better for a family if the father works outside the home and the mother takes care of children." 51% of the sampled women agreed with the statement while 57% of the sampled men agreed. A note on the polling method says that about 1600 men and 1800 women were randomly sampled in the poll and the margin of error was about two percentage points. What is the margin of error and how is it interpreted in this context?*	Conceptual Procedural Application	Communicating Reasoning Modeling & Data Analysis
ST.S-IC.IA.B.1 Conduct statistical investigations. ★ a. Conduct observational studies. b. Conduct statistical experiments.	Conceptual Procedural Application	Problem Solving Communicating Reasoning Modeling & Data Analysis

Domain: ST.S-CP Conditional Probability and Rules of Probability ★

ST.S-CP.A Cluster: Understand independence and conditional probability and use them to interpret data.★

Standard	Rigor	SMP Bundle
ST.S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or" "and" "not.") ★	Conceptual Procedural Application	Problem Solving
ST.S-CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent. ★	Conceptual Procedural Application	Problem Solving
ST.S-CP.A.3 Understand the conditional probability of A given B as $\frac{P(A\ and\ B)}{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★	Conceptual Procedural Application	Problem Solving
ST.S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* ★	Conceptual Procedural Application	Problem Solving
ST.S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* ★	Conceptual Procedural Application	Problem Solving

ST.S-CP.B Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Standard	Rigor	SMP Bundle
ST.S-CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model. ★	Conceptual Procedural Application	Problem Solving
ST.S-CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model. ★	Conceptual Procedural Application	Problem Solving
ST.S-CP.B.8 Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B)P(B)|A) = P(B)P(A|B), and interpret the answer in terms of the model.★	Conceptual Procedural Application	Problem Solving
ST.S-CP.B.9 Use permutations and combinations to compute probabilities of compound events and solve problems. ★	Conceptual Procedural Application	Problem Solving

Domain: ST.S-MD Using Probability to Make Decisions ★

ST.S-MD.A Cluster: Calculate expected values and use them to solve problems.

Standard	Rigor	SMP Bundle
ST.S-MD.A.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. ★	Conceptual Procedural Application	Problem Solving
ST.S-MD.A.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ★	Conceptual Procedural Application	Problem Solving
ST.S-MD.A.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* ★	Conceptual Procedural Application	Problem Solving

Standard	Rigor	SMP Bundle
ST.S-MD.A.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* ★	Conceptual Procedural Application	Problem Solving

ST.S-MD.B Cluster: Use probability to evaluate outcomes of decisions.

Standard	Rigor	SMP Bundle
ST.S-MD.B.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. ★ a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* b. Evaluate and compare strategies based on expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*	Conceptual Procedural Application	Problem Solving
ST.S-MD.B.6 Use probabilities to make fair decisions. *For example, drawing by lots, using a random number generator.* ★	Conceptual Procedural Application	Problem Solving
ST.S-MD.B.7 Analyze decisions and strategies using probability concepts. *For example, product testing, medical testing, pulling a hockey goalie at the end of a game.* ★	Conceptual Procedural Application	Problem Solving

Conceptual Category: Statistics and Probability ★

Domain: N-APR Arithmetic with Polynomials and Rational Expressions

N-APR.C: Use polynomial identities to solve problems.

Standard	Rigor	SMP Bundle
ST.A-APR.C.5 Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. *For example, constructing a distribution for the number made for a 35% free-throw shooter when given 10 attempts.*	Conceptual Procedural	Problem Solving Communicating Reasoning

Precalculus

Narrative

The precalculus course is one of the courses that students may choose as part of the pathway that prepares for technical careers as well as general college majors like life science, health, business and social science.

In precalculus, students extensively explore functions, mastering their properties and applications. They identify and graph various function types, such as linear, quadratic, polynomial, rational, exponential, logarithmic, and trigonometric functions, deepening their understanding of function characteristics like domain, range, and symmetry. Additionally, students develop procedural skills in performing operations and solving equations involving these functions, applying their knowledge to real-world scenarios. Students comprehensively understand function analysis and its significance in mathematical reasoning and problem solving through conceptual understanding and procedural practice.

Additionally, students define exponential and logarithmic functions and their properties, compose functions, and graph and analyze their transformations. Students apply their knowledge to practical situations by modeling data sets with exponential functions and scenarios with logarithmic functions, strengthening their modeling and data analysis skills. Throughout the unit, students identify assumptions and limitations of function models, fostering critical thinking and conceptual understanding. Students comprehensively understand functions and their significance in mathematical reasoning and problem solving through conceptual exploration, procedural practice, and real-world application.

In precalculus, students explore trigonometric functions and their applications, mastering their properties and graphing techniques. They define trigonometric functions, gaining insight into their behavior by graphing functions such as sine, cosine, tangent, cosecant, secant, and cotangent. Additionally, students use the unit circle to define trigonometric values, enhancing their understanding of trigonometric relationships. They apply inverse trigonometric functions to solve equations and inequalities, tackling various problem-solving scenarios. Students improve their modeling and analysis skills by modeling data with sinusoidal functions and using their knowledge in real-world situations.

Moreover, students explore graphing functions using polar coordinates, describing the dynamic relationship between angles and radii in polar graphs. They also employ trigonometric identities to simplify expressions, verify identities, and solve equations using algebraic and graphical methods. Through application problems involving triangles, vectors, and periodic phenomena, students apply trigonometric identities to solve complex problems, reinforcing their understanding of trigonometric concepts and their practical applications. Students comprehensively understand trigonometric functions and their significance in mathematical reasoning and problem solving through conceptual exploration, procedural practice, and real-world application.

In precalculus, students explore conic sections, including circles, parabolas, ellipses, and hyperbolas, mastering their properties and applications. They develop proficiency in graphing conic sections, solving related problems, and applying transformations, enhancing their analytical skills. Through conceptual and procedural practice, students comprehensively understand conic sections and their significance in mathematical reasoning and problem solving.

In precalculus, students delve into sequences, series, and parametric functions, mastering their properties and applications. They define arithmetic and geometric sequences and series, analyze mathematical structures, and apply them to solve real-world problems. Additionally, students explore parametric functions, graphing conic sections, and describing the motion of objects using vectors and transformation matrices. Through conceptual understanding and procedural practice, students comprehensively understand these mathematical concepts and their significance in mathematical reasoning and problem solving across different disciplines.

Note: Cluster level emphasis (for example, Major, Supporting, and Additional designations) for this course are intentionally not included because the content is beyond the college and career readiness level for all students.

Precalculus Overview

Clusters are listed below. The code, PC. F-PREL denotes precalculus **course**, functions **conceptual category**, and operations with polynomial, rational, exponential, and logarithmic functions **domain.**

Operations with Polynomial, Rational, Exponential, and Logarithmic Functions (F-PREL)

- Identify, graph, analyze functions and perform function operations.
- Analyze, graph, and solve problems using polynomial and rational functions.
- Analyze, graph, and solve problems using exponential and logarithmic functions.

Properties of Trigonometric Functions (F-TF)

- Analyze, graph, and solve problems using trigonometric functions.
- Use trigonometric identities to solve problems.

Analytic Geometry (F-AG)

- Solve problems using properties of analytic geometry.

Reasoning with Sequences and Series (F-SS)

- Solve problems involving sequences and series.
- Reason with functions involving parameters, vectors, and matrices.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Conceptual Category: PC.F Functions

Domain: PC.F-PREL Operations with Polynomial, Rational, Exponential, and Logarithmic Functions

PC.F-PREL.A: Identify, graph, analyze functions and perform function operations.

Standard	Rigor	SMP Bundle
PC.F-PREL.A.1 Understand the concept of a function and its notation.	Conceptual	Communicating Reasoning
PC.F-PRELA.2 Identify and graph various functions, including linear, quadratic, polynomial, rational, exponential, logarithmic, and trigonometric functions.	Conceptual Procedural	Communicating Reasoning
PC.F-PREL.A.3 Analyze functions by considering domain, range, symmetry, intercepts, and asymptotic behavior.	Conceptual Procedural	Communicating Reasoning
PC.F-PREL.A.4 Perform operations of addition, subtraction, multiplication, division, and composition of functions.	Procedural	Communicating Reasoning

PC.F-PREL.B: Analyze, graph, and solve problems using polynomial and rational functions.

Standard	Rigor	SMP Bundle
PC.F-PREL.B.1 Describe how quantities change with respect to each other.	Conceptual	Communicating Reasoning
PC.F-PREL.B.2 Understand the behavior of polynomial functions, including end behavior, turning points, and factors.	Conceptual	Communicating Reasoning
PC.F-PREL.B.3 Perform polynomial long division and synthetic division.	Procedural	Communicating Reasoning
PC.F-PREL.B.4 Identify and graph rational functions and analyze their asymptotic behavior and discontinuities.	Conceptual Procedural	Communicating Reasoning
PC.F-PREL.B.5 Solve polynomial and rational equations and inequalities.	Procedural	Problem Solving
PC.F-PREL.B.6 Model aspects of scenarios using polynomial and rational functions.	Procedural Application	Modeling & Data Analysis
PC.F-PREL.B.7 Identify assumptions and limitations of function models.	Conceptual Procedural	Communicating Reasoning

PC.F-PREL.C: Analyze, graph, and solve problems using exponential and logarithmic functions.

Standard	Rigor	SMP Bundle
PC.F-PREL.C.1 Define exponential and logarithmic functions and their properties.	Conceptual	Communicating Reasoning
PC.F-PREL.C.2 Compose exponential and logarithmic functions and find inverses.	Procedural	Problem Solving
PC.F-PREL.C.3 Graph exponential and logarithmic functions and understand their transformations.	Procedural	Problem Solving Communicating Reasoning
PC.F-PREL.C.4 Solve exponential and logarithmic equations and inequalities.	Procedural	Problem Solving
PC.F-PREL.C.5 Apply exponential and logarithmic functions in real-world contexts such as population growth, compound interest, and exponential decay.	Procedural Application	Problem Solving
PC.F-PREL.C.6 Model data sets with exponential functions.	Procedural Application	Modeling & Data Analysis
PC.F-PREL.C.7 Model scenarios with logarithmic functions.	Procedural Application	Modeling & Data Analysis

Domain: PC.F-TF Properties of Trigonometric Functions

PC.F-TF.A: Analyze, graph, and solve problems using trigonometric functions.

Standard	Rigor	SMP Bundle
PC.F-TF.A.1 Define trigonometric functions and their reciprocal functions.	Conceptual	Communicating Reasoning
PC.F-TF.A.2 Graph trigonometric functions including sine, cosine, tangent, cosecant, secant, and cotangent.	Procedural	Problem Solving
PC.F-TF.A.3 Understand the unit circle and use it to define trigonometric values for unique angles.	Conceptual	Communicating Reasoning
PC.F-TF.A.4 Use inverse trigonometric functions to solve trigonometric equations.	Procedural	Problem Solving
PC.F-TF.A.5 Solve trigonometric equations and inequalities and apply trigonometric identities.	Procedural	Problem Solving
PC.F-TF.A.6 Model data and scenarios with sinusoidal functions.	Procedural Application	Modeling & Data Analysis
PC.F-TF.A.7 Graphing functions using polar coordinates.	Procedural	Problem Solving
PC.F-TF.A.8 Describe how angles and radii change with respect to each other in a polar graph.	Conceptual	Communicating Reasoning

PC.F-TF.B: Use trigonometric identities to solve problems.

Standard	Rigor	SMP Bundle
PC.F-TF.B.1 Use trigonometric identities to simplify expressions and verify identities.	Procedural	Problem Solving
PC.F-TF.B.2 Solve trigonometric equations using algebraic and graphical methods.	Procedural	Problem Solving
PC.F-TF.B.3 Apply trigonometric identities to solve problems involving triangles, vectors, and periodic phenomena.	Procedural Application	Problem Solving

Domain: PC.F-AG Analytic Geometry

PC.F-AG.A: Solve problems using properties of analytic geometry.

Standard	Rigor	SMP Bundle
PC.F-AG.A.1 Understand the properties of conic sections, including circles, parabolas, ellipses, and hyperbolas.	Procedural Application	Problem Solving
PC.F-AG.A.2 Graph conic sections in standard and general forms.	Procedural	Problem Solving
PC.F-AG.A.3 Solve problems involving distance, midpoint, slope, and equations of lines and circles.	Procedural	Problem Solving
PC.F-AG.A.4 Use transformations to analyze and graph geometric figures.	Procedural	Problem Solving

Domain: PC.F-SS Reasoning with Sequences and Series

PC.F-SS.A: Solve problems involving sequences and series.

Standard	Rigor	SMP Bundle
PC.F-SS.A.1 Define arithmetic and geometric sequences and series.	Conceptual	Communicating Reasoning
PC.F-SS.A.2 Find the nth term, partial sums, and sums of finite and infinite sequences and series.	Procedural	Problem Solving
PC.F-SS.A.3 Apply sequences and series to solve problems in mathematics and other disciplines.	Procedural Application	Problem Solving

PC.F-SS.B: Reason with functions involving parameters, vectors, and matrices.

Standard	Rigor	SMP Bundle
PC.F-SS.B.1 Describe how quantities change with respect to each other in a parametric function.	Conceptual	Communicating Reasoning
PC.F-SS.B.2 Graph conic sections using implicitly defined functions and parametric functions.	Procedural	Problem Solving
PC.F-SS.B.3 Use vectors to describe the motion of an object.	Procedural	Problem Solving
PC.F-SS.B.4 Describe the impact of a transformation matrix on a graphical object.	Conceptual	Communicating Reasoning
PC.F-SS.B.5 Model change in a context using matrices.	Procedural Application	Modeling & Data Analysis

Calculus

The calculus course is one of the courses that students may choose as part of the pathway that prepares for technical careers as well as general college majors like life science, health, business and social science.

In calculus, students explore the crucial concept of limits, mastering their definitions and diverse evaluation methods. They also understand how limits define continuity and apply these principles to analyze functions, using tools like the Intermediate Value Theorem. Students develop a deep understanding of limits and continuity through rigorous study and problem solving. For instance, they can apply these concepts to real-world scenarios, such as predicting the maximum speed of a car on a racetrack or analyzing the rate of change in a company's stock prices. These examples demonstrate the practical relevance of calculus in various fields.

In calculus, students explore differentiation and its applications. They identify derivatives through various strategies by estimating derivatives and identifying instantaneous rates of change. Additionally, they study continuity, compute derivatives of algebraic and trigonometric functions, and apply rules like the product and chain rules. Through analysis and problem solving, they find critical points, solve optimization problems, and interpret derivatives in real-world contexts. For example, when finding the derivative of a complex function, they can break it down into simpler parts and apply the chain rule step by step, making the process more manageable and understandable.

Students explore integration and its applications. They utilize definite integrals to calculate net change and approximate integrals with finite Riemann Sums. Understanding integrals as change measures, they apply the Fundamental Theorem of Calculus to evaluate and construct antiderivatives. Students compute indefinite and definite integrals of various functions by employing techniques like substitution and integration by parts. They also analyze convergence or divergence of improper integrals and apply integration to solve problems in physics and economics, demonstrating its versatility in real-world contexts.

In calculus, students explore differential equations and their real-world applications. They understand the concept's significance in modeling phenomena and solve first-order equations, including growth and decay models. Through this, they interpret solutions within real-world contexts, gaining insight into predicting changes in various systems. Students deepen their understanding of calculus and its practical applications in modeling dynamic systems by applying differential equations to scenarios.

Students cover more topics like sequences, series, Taylor polynomials, and polar coordinates, enhancing their understanding of calculus concepts and applications. Through interdisciplinary problem solving, they gain practical experience applying calculus to real-world scenarios, strengthening their foundational knowledge and critical thinking skills, preparing them for further study in mathematics, science, engineering, and related fields.

Technology should be used regularly to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to help in interpreting results.

Note: Cluster level emphasis (for example, Major, Supporting, and Additional designations) for this course are intentionally not included because the content is beyond the college and career readiness level for all students.

Calculus Overview

Clusters are listed below. The code, C.F-LC denotes calculus **course**, functions conceptual category, and limits and continuity **domain.**

Limits and Continuity (F-LC)

- Compute limits of functions.
- Solve problems involving continuity determine the continuity of functions.

Derivatives (F-D)

- Use limits to compute derivatives.
- Compute derivatives.
- Apply the concept of the derivative.

Integrals (F-I)

- Define and interpret integrals.
- Compute integrals.
- Use integrals to solve problems.

Differential Equations (F-DE)

- Solve and interpret solutions of differential equations.

Advanced Topics (F-AT)

- Apply calculus and related skills to advanced topics.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Conceptual Category: C.F Functions

Domain: C.F-LC Limits and Continuity

C.F-LC.A: Compute limits of functions.

Standard	Rigor	SMP Bundle
C.F-LC.A.1 Define limits and explain their significance in calculus.	Conceptual	Communicating Reasoning
C.F-LC.A.2 Evaluate limits algebraically, graphically, and numerically.	Conceptual	Communicating Reasoning

C.F-LC.B: Solve problems involving continuity and determine the continuity of functions.

Standard	Rigor	SMP Bundle
C.F-LC.B.1 Use limits to define continuity at a point.	Conceptual	Communicating Reasoning
C.F-LC.B.2 Apply the concepts of continuity and the Intermediate Value Theorem.	Conceptual Application	Communicating Reasoning

Domain: C.F-D Derivatives

C.F-DF.A: Use limits to compute derivatives.

Standard	Rigor	SMP Bundle
C.F-D.A.1 Estimate derivatives using difference quotients.	Procedural	Problem Solving
C.F-D.A.2 Define the instantaneous rate of change at a point as the limit of average rates of change.	Procedural	Problem Solving
C.F-D.A.3 Identify the derivative of a function using appropriate strategies. *For example, rules for sums, differences, products, quotients, and limits of functions*.	Procedural Conceptual	Problem Solving Communicating Reasoning
C.F-D.A.4 Interpret derivatives as instantaneous rates of change.	Procedural Conceptual	Problem Solving
C.F-D.A.5 Explain the relationship between continuity and differentiability at a point.	Conceptual	Communicating Reasoning
C.F-D.A.6 Use limits to define the derivative function.	Procedural	Problem Solving
C.F-DF.A.7 Use limits to find derivatives of simple algebraic functions.	Procedural	Problem Solving

C.F-D.B: Compute derivatives.

Standard	Rigor	SMP Bundle
C.F-D.B.1 Compute derivatives of algebraic, trigonometric, inverse, exponential, and logarithmic functions.	Procedural	Problem Solving
C.F-D.B.2 Apply the rules of differentiation, including the product rule, quotient rule, and chain rule.	Procedural Application	Problem Solving
C.F-D.B.3 Compute derivatives of general inverse and implicitly defined functions.	Procedural	Problem Solving
C.F-D.B.4 Compute higher-order derivatives.	Procedural	Problem Solving

C.F-DF.C: Apply the concept of the derivative.

Standard	Rigor	SMP Bundle
C.F-D.C.1 Find the slope of a tangent line at a point.	Procedural	Problem Solving
C.F-D.C.2 Determine analytically and graphically where a function, or derivative function, is positive or negative, increasing or decreasing, and/or concave up or concave down.	Procedural	Problem Solving
C.F-D.C.3 Explain the relationships among the behaviors of $f, f', and\ f''$; for example, if f'' is positive, then f' is increasing and f is concave up.	Procedural	Problem Solving
C.F-D.C.4 Analyze and sketch graphs of $f, f', and\ f''$.	Procedural Conceptual	Communicating Reasoning
C.F-D.C.5 Use the first and second derivative tests to find and classify critical points.	Procedural Conceptual	Problem Solving Communicating Reasoning
C.F-D.C.6 Apply the Mean Value Theorem and Extreme Value Theorem.	Procedural	Problem Solving
C.F-D.C.7 Solve optimization problems involving maxima and minima.	Procedural	Problem Solving
C.F-D.C.8 Solve related rate problems.	Procedural	Problem Solving
C.F-D.C.9 Use L'Hospital's rule to compute limits.	Procedural	Problem Solving
C.F-D.C.10 Interpret derivatives in real-world contexts, such as motion and growth problems.	Procedural Conceptual	Problem Solving Communicating Reasoning

Domain: C.F-I Integrals

C.F-I A: Define and interpret integrals.

Standard	Rigor	SMP Bundle
C.F-I.A.1 Use definite integrals to determine net change over an interval.	Conceptual	Problem Solving
C.F-I.A.2 Approximate definite integrals using finite Riemann Sums.	Procedural Conceptual	Problem Solving Communicating Reasoning
C.F-I.A.3 Define the definite integral as a limit of a finite Riemann Sum.	Procedural	Problem Solving
C.F-I.A.4 Interpret definite integrals as net change.	Procedural Conceptual	Problem Solving Communicating Reasoning
C.F-I.A.5 State the Fundamental Theorem of Calculus and use it to evaluate definite integrals and construct antiderivatives.	Procedural	Problem Solving

C.F-I.B: Compute integrals.

Standard	Rigor	SMP Bundle
C.F-I.B.1 Compute indefinite and definite integrals of algebraic, trigonometric, exponential, inverse, and logarithmic functions.	Procedural	Problem Solving
C.F-I.B.2 Apply integration techniques, including substitution and integration by parts.	Procedural	Problem Solving
C.F-I.B.3 Determine if an improper integral converges or diverges using limits of definite integrals.	Procedural Conceptual	Problem Solving
C.F-I.B.4 Transform integrands (using substitution and other techniques) to find antiderivatives using a table of integrals.	Procedural Conceptual	Problem Solving

C.F-I.C: Use integrals to solve problems.

Standard	Rigor	SMP Bundle
C.F-I.C.1 Solve problems involving area, volume, and average value of functions.	Procedural	Problem Solving
C.F-I.C.2 Use integrals to find the area between curves and volume of solids of revolution.	Procedural Conceptual	Problem Solving Communicating Reasoning
C.F-I.C.3 Use integrals to solve problems in physics, economics, and other fields.	Procedural	Problem Solving

Domain: C.F-DE Differential Equations

C.F-DE.A: Solve and interpret solutions of differential equations.

Standard	Rigor	SMP Bundle
C.F-DE.A.1 Explain the concept of a differential equation.	Conceptual	Communicating Reasoning
C.F-DE.A.2 Solve first-order differential equations, including linear and exponential growth and decay models.	Conceptual	Communicating Reasoning
C.F-DE.A.3 Interpret solutions of differential equations in real-world contexts.	Procedural	Problem Solving

Domain: C.AT Advanced Topics

C.F-AT.A: Apply calculus and related skills to advanced topics.

Standard	Rigor	SMP Bundle
C.F-AT.A.1 Explore additional topics such as sequences and series, Taylor polynomials, and polar coordinates (optional depending on course length and student readiness).	Conceptual	Communicating Reasoning
C.F-AT.A.2 Apply calculus concepts to interdisciplinary problems and projects.	Conceptual	Problem Solving
C.F-AT.A.3 Prepare for further study in mathematics, science, engineering, and related fields.	Conceptual	Communicating Reasoning

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1,000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0–5, 0–10, 0–20, or 0–100, respectively. Example: $8 + 2 = 10$ is an addition within 10, $14 - 5 = 9$ is a subtraction within 20, and $55 - 18 = 37$ is a subtraction within 100. Addition and subtraction within 20 involve single-digit addends only.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $\left(\frac{3}{4}\right)$ and $\left(-\frac{3}{4}\right)$ are additive inverses of one another because $\frac{3}{4} + \left(-\frac{3}{4}\right) = \left(-\frac{3}{4}\right) + \left(\frac{3}{4}\right) = 0$.

Algorithm. A step-by-step procedure used for solving a problem or performing a calculation.

Aspects of Rigor. The third shift consists of conceptual understanding, procedural skills, fluency, and application.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

Cluster. Groups of related standards.

Coherence. Coherence is the second of the 3 Mathematical Shifts (see Mathematical Shifts in glossary). This shift requires connections. within and across grade levels to ensure students experience a logical mathematical progression.

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction $\frac{A}{B}$ where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Conceptual category. Categories for high school standards that span across multiple courses.

Congruent. Two planes or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Conjecture. Conclusion about something based on how it seems and not on proof.

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. *For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."*

Curriculum. Organized plan of instruction comprised of a sequence of instructional units that engages students in mastering the standards.

Decimal. A fraction whose denominator is a power of ten and whose numerator is expressed by figures placed to the right of a decimal point.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center and multiplies distances from the center by a common scale factor.

Domain. Groups of related clusters.

Dot plot. See: line plot.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. *For example,* $643 = 600 + 40 + 3$.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

Fluency. Ability to apply procedures efficiently, flexibly, and accurately including fact fluency, computational fluency, and procedural fluency.

Focus. Focus is the first of the 3 Mathematical Shifts. (see Mathematical shifts in the glossary). This shift emphasizes prioritizing the Major Work of each grade level. 65–85% of class time is devoted to the Major Work of the grade.

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number a.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.

Mathematical Shifts. The three shifts in mathematics are focus, coherence, and rigor. (see definitions of focus, coherence, and rigor) These shifts provide a frame that describes how the standards raise expectations across multiple areas of students' educational experience including instructional materials, classroom practice, and assessment.

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2,3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2,3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line is halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0–100. Example: $72 \div 8 = 9$. Multiplication and division within 100 involve single-digit factors only. Multiplication and division within 100 involve single-digit factors only.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$.

Number line diagram. A diagram of the number line is used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Number sense. refers to a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations. (Reys & Yang, 1998; McIntosh al., 1999).

Objective or Outcome. What a student can do as a result of completing a learning experience (observable and measurable).

Percent rate of change. A rate of change expressed as a percentage. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.

Progressions. Narrative documents describing the development of a topic across several grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or -a/b for some fraction a/b. Rational numbers include integers, which are a subset of all rational numbers.

Rectilinear figure. a polygon in which all angles are right angles.

Repeating decimal. The decimal form of a rational number. See also: terminating decimal.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Rigor. Rigor is the third of the 3 Mathematical Shifts (see Mathematical Shifts in glossary). This shift pursues conceptual understanding, procedural skill and fluency, and application with equal intensity,

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. *For example, the heights and weights of a group of people could be displayed on a scatter plot.*

Similarity transformation. A rigid motion followed by a dilation.

Standard. Broad learning goals articulating what students should know, understand and be able to do over a given time.

Standard Algorithm. The standard algorithm represents an efficient and universally applicable method of adding, subtracting, multiplying, and dividing.

Strategies. Flexible approaches to solve a problem using properties of operations and place value.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

Appendix A: Tables

Table 1. Common addition and subtraction situations. [1]

	Results Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown1**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2. Common multiplication and division situations.[6]

	Unknown Product 3 × 6 = ?	Group Size Unknown ("How many in each group?" Division) 3 × ? = 18, and 18 ÷ 3 = ?	Number of Groups Unknown ("How many groups?" Division) ? × 6 = 18, and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? Area example. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a × b = ?	a × ? = p, and p ÷ a = ?	? × b = p, and p ÷ b = ?

[4] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[6] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	*For every a there exists –a so that* $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	*For every* $a \neq 0$ *there exists* $1/a$ *so that* $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	*If* $a = b$, *then* $b = a$.
Transitive property of equality	*If* $a = b$ *and* $b = c$, *then* $a = c$.
Addition property of equality	*If* $a = b$, *then* $a + c = b + c$.
Subtraction property of equality	*If* $a = b$, *then* $a - c = b - c$.
Multiplication property of equality	*If* $a = b$, *then* $a \times c = b \times c$.
Division property of equality	*If* $a = b$ *and* $c \neq 0$, *then* $a \div c = b \div c$.
Substitution property of equality	*If* $a = b$, *then b may be substituted for a in any expression containing a.*

Table 5. The properties of inequality. Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.

If $a > b$ *and* $b > c$ *then* $a > c$.

If $a > b$, *then* $b < a$.

If $a > b$, *then* $-a < -b$.

If $a > b$, *then* $a \pm c > b \pm c$.

If $a > b$ *and* $c > 0$, *then* $a \times c > b \times c$.

If $a > b$ *and* $c < 0$, *then* $a \times c < b \times c$.

If $a > b$ *and* $c > 0$, *then* $a \div c > b \div c$.

If $a > b$ *and* $c < 0$, *then* $a \div c < b \div c$.

Appendix B: Works Consulted

ACT College Readiness Benchmarks™

ACT College Readiness Standards™

ACT National Curriculum Survey™

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