HS Target N

Domain, Target, Standards, DOK, Vertical Alignments, Achievement Levels, Evidence Required, Vocabulary, Response Types, Materials, Attributes, Question Types, and Question Banks (Examples)

Content Domain: Functions

Target N [m]: F.BF.A Build a function that describes a relationship between two quantities.

Standards included in Target N: F-BF.A, F-BF.A.1 F-BF.A.1a, F-BF.A.2

F.BF.A Build a function that describes a relationship between two quantities.

F-BF.A.1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Vertical Alignment

8.F.A Define, evaluate, and compare functions.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.B Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Achievement Level Descriptors

Level 1 Students should be able to identify an explicit or a recursive function and determine the steps for calculation from a context requiring up to two steps. They should be able to add and subtract two linear functions.

Level 2 Students should be able to build an explicit or a recursive function to describe or model a relationship between two quantities and determine the steps for calculation from a context. They should be able to add, subtract, and multiply linear and quadratic functions.

Level 3 Students should be able to translate between explicit and recursive forms of a function. They should be able to add, subtract, multiply, and divide functions.

Level 4 Students should be able to determine when it is appropriate to combine functions using arithmetic operations in context.

Evidence Required

1. The student writes explicit or recursive functions to describe relationships between two quantities from a context.

2. The student translates between explicit formulas and recursively defined functions.

3. The student understands a function as a model of the relationship between two quantities.

Vocabulary

Function(s), quantity, quantities, explicit, recursive, arithmetic sequence, geometric sequence, input, output, ordered pairs

Response Types

Multiple Choice, single correct response; Equation/Numeric; Matching Table; Fill-in Table

Materials

The student is presented with a contextual situation familiar to 16–17 year olds where a function can describe a relationship between two quantities. Contextual situations will be introduced with simple subject-verb-object sentences, avoiding long noun phrases, multiple prepositional phrases, and unfamiliar technical vocabulary. Specific stimuli include: explicit functions, recursive functions, written descriptions of functional relationships between two quantities, a sequence of numbers, a table representing a sequence of numbers and corresponding term numbers, a sequence in which the first four numbers are given or any four consecutive terms excluding a_1.

Attributes

When translating between an explicit function and a recursive function, functions are limited to arithmetic and geometric relations.

Claim 1: Concepts and Procedures (DOK 1) Question Banks

Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Claim 1 F-BF.A.1 DOK Level 2

Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Evidence Required

The student writes explicit or recursive functions to describe relationships between two quantities from a context.

Question Type 1: The student is presented with a contextual situation.

1. Maria is making a rectangular garden. The length of the garden is 2 yards greater than its width, w, in yards.

Enter the function, f(w), that describes the area, in square yards, of Maria's garden as a function of the width, w.

2. Barb traveled 300 miles during the first 5 hours of her trip. Barb then traveled at a constant speed of 50 miles per hour for the remainder of the trip.

Enter the function, f(t), that describes the average speed during the entire trip as a function of time, t, in hours, Barb traveled after her first 300 miles.

3. A washing machine was purchased for \$256. Each year the value is $\frac{1}{4}$ of its value the previous year.

Enter the function, f(t), that describes the value of the washing machine, in dollars, as a function of time in years, t, after the initial purchase.

Rubric: (1 point) The student correctly enters the function describing the relationship between two quantities in the given contextual situation (e.g., $f(w) = w(w+2)$; $f(\hbar)=300+50\hbar / 5+\hbar$; $f(t) = \$256(0.75)^{t}$).

Response Type: Equation/Numeric

Question Type 2: The student is presented with a contextual situation.

1. A researcher studies the growth of a fruit fly population. The researcher counts the number of fruit flies at noon each day. The results are in the table.

Day	Number of Fruit Flies
0	4
1	8
2	16
3	32

- $V(n)$= Total number of fruit flies after n days
- $V(0)=4$

Enter the function for $n\geq1$, which describes the number of fruit flies, $V(n)$, at noon on the nnth day in terms of the number of fruit flies at noon on the previous day, $V(n-1)$.

Rubric: (1 point) Student correctly enters the function describing the relationship between two quantities in the given contextual situation [e.g., V(n) = 2V(n – 1)].

Response Type: Equation/Numeric

Question Type 3: The student is presented with a contextual situation.

1.The first row in a theater has 8 seats, the second row has 11 seats, the third row has 14 seats and the fourth row has 17 seats.

The pattern of increasing each successive row by 3 seats continues throughout the theater.

- $f(r)$ = the number of seats in row r.
- $f(1)=8$ Enter an equation, for $r\geq2$, which describes the number of seats, $f(r)$, in the rth row in terms of the number of seats in the $(r - 1)$th row, $f(r - 1)$.

2. The 13th row in a theater has 41 seats, the 12th row has 38 seats, the 11th row has 35 seats and the 10th row has 32 seats. The pattern of decreasing each successive row by 3 seats continues from the 13th row to the 1st row.

- $f(r)$ = the number of seats in row rr.

• $f(1)=5$ Enter an equation, for $r \geq 2$, that describes the number of seats, $f(r)$, in the rrth row in terms of the number of seats in the $(r-1)$th row, $f(r-1)$. Assume that the pattern described applies to all rows.

Rubric: (1 point) Student correctly represents the sequence with a recursively defined function [e.g., $f(r)=f(r-1)+3$; $f(r)= f(r-1)+3$].

Response Type: Equation/Numeric

Claim 1 F-BF.A.2 DOK Level 2

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Evidence Required
The student translates between recursive functions and explicit functions.

Question Type 1: The student is presented with an explicit or recursively defined function.

1. Consider this function in explicit form.

$f(n)=3n-4$; $n \geq 1$

Select the equivalent recursively defined function.

A. $f(1)=-1$ $ff(n)=f(n-1)+3$; $n \geq 2$
B. $f(1)=-1$ $f(n)=3f(n-1)$; $n \geq 2$
C. $f(0)=-4$ $f(n)=3f(n-1)$; $n \geq 2$
D. $f(0)=-4$ $f(n)=f(n-1)+3$; $n \geq 2$

Question Type 2: Consider this function in recursive form.

$f(1)=-3$
$f(n)=3f(n-1)$; $n \geq 2$

Select the equivalent explicit function for $nn \geq 1$.

A. $f(n)=-3(n)$
B. $f(n)=-3(n-1)$
C. $f(n)=-3(3)n$
D. $f(n)=-3(3)(n-1)$

Rubric: (1 Point) Student selects the correct choice (e.g., A; D).

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Response Type: Multiple Choice, single correct response

Question Type 3: The student is presented with explicit and recursive functions.

1. The functions in the table are defined for integers $n \geq 1$.

Match each recursively defined function with the equivalent explicit form.

Functions	$f(n) = 3(10)^{(n-1)}$; $n \geq 1$	$f(n) = 3n + 7$; $n \geq 1$	$f(n) = 10(3)^{(n-1)}$; $n \geq 1$
$f(1) = 10$ $f(n) = 3f(n-1)$; $n \geq 2$			
$f(1) = 3$ $f(n) = 10f(n-1)$; $n \geq 2$			
$f(1) = 10$ $f(n) = f(n-1) + 3$; $n \geq 2$			

Click the appropriate box that matches the recursive form in the first column with its equivalent explicit form in the top row.

Interaction: The student is presented with three explicit functions in the first row and three recursive functions in the first column. The student selects the cell in the table that matches the functions.

Rubric: (1 point) Student correctly matches all functions (e.g., see below).

Functions	$f(n) = 3(10)^{(n-1)}$; $n \geq 1$	$f(n) = 3n + 7$; $n \geq 1$	$f(n) = 10(3)^{(n-1)}$; $n \geq 1$
$f(1) = 10$ $f(n) = 3f(n-1)$; $n \geq 2$			
$f(1) = 3$ $f(n) = 10f(n-1)$; $n \geq 2$			
$f(1) = 10$ $f(n) = f(n-1) + 3$; $n \geq 2$			

Response Type: Matching Tables
Claim 1 F-BF.A.1 DOK Level 2

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Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Evidence Required
The student understands a function as a model of the relationship between two quantities.

Question Type 1: The student is presented with a contextual situation.

1. A theater needs to place seats in rows. The function, f(r), as shown below, can be used to determine the number of seats in each row, where r is the row number.

$f(1)=8$
$f(r)=f(r-1)+3$

Use the function to complete the table indicating the number of seats in each of the first four rows of the theater.

Row number	Number of Seats
Row 1	
Row 2	
Row 3	
Row 4	

Rubric: (1 point) Student correctly enters the sequence from the recursive form into the table (e.g., see below).

Row number	Number of Seats
Row 1	8
Row 2	11
Row 3	14
Row 4	17

Response Type: Fill-in Table

Claim 2 Problem Solving Questions Banks
Claim Descriptors and Targets

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

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Example 1

Susan has an ear infection. Her doctor prescribes an antibiotic. The doctor tells Susan to take a 250-milligram dose of the antibiotic every 12 hours for the next 10 days.

- Susan finds out that 4% of the antibiotic is still in her body after 12 hours.
- Assume that each dose is exactly 250 milligrams and that Susan takes one dose every 12 hours.

Part A
How much of the antibiotic, in milligrams, is in Susan's body immediately after taking the 2nd dose? Enter your answer in the first response box.

Part B
How much of the antibiotic, in milligrams, is in Susan's body immediately after taking the 10th dose? Enter your answer in the second response box.

Rubric: (2 points) The student enters the correct amount of antibiotic in Susa's body for Part A and B (e.g., 260, 260.4167). Note: An acceptable range for Part B is 260.4-260.422.
(1 point) The student enters the correct amount for Part A or Part B, but not both.

Response Type: Equation/Numeric (2 response boxes)

Example 2

P(x) represents the cost that an online bookstore charges for shipping items and packaging material that together weigh x pounds. The packaging material weighs 1 pound.

A competitor charges the same rate per pound but does not charge for the weight of the packaging material.
However, the competitor does charge an additional $5 processing fee for each shipment.

Which expression represents the cost of shipping x pounds with the competitor?

A. $P(x + 5) + 1$
B. $P(x + 5) - 1$
C. $P(x + 1) + 5$
D. $P(x - 1) + 5$

Rubric: (1 point). The student selects the correct answer choice (D).

Response Type: Multiple choice, single correct response.

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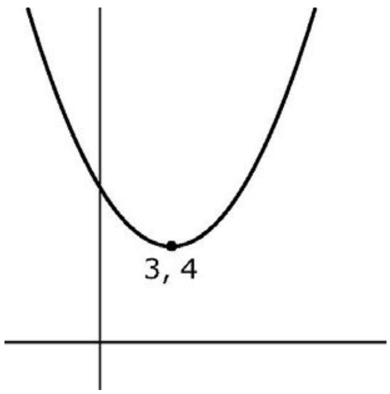
Claim 3 Communicating Reasoning Question Banks
[Claim Descriptors and Targets]

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Example 1

The graph of a quadratic function f is shown and the vertex is labeled with its coordinates.
If g(x) = f(x-1)+2, what is the minimum value of g?



The minimum value of g is [1, 2, 3, 4, 5, 6] because the minimum value for f is [1, 2, 3, 4, 5, 6] and the graph of g is shifted [up, down, left, right] from the graph of f by [1, 2, 3, 4, 5, 6] units (in addition to the other shift).

Rubric: (1 points) The student selects the correct choices from the drop-down menus (6; 4; up; 2 or 6; 4; right; 1).

Response Type: Drop Down Menu

Example 2
An arithmetic sequence is sequence in which the difference between any two consecutive terms is the same.

For example, the sequence 2, 9, 16, 23, 30, 37 is an arithmetic sequence because the difference between any two consecutive numbers is 7.

Suppose that an arithmetic sequence of integers starts with a 5 and also later includes an 11.
5, …?.... , 11, ____

Which number could not be the term that immediately follows 11 in the sequence?
A. 12
B. 13
C. 14

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D. 15
E. 17

Rubric: (1 point) The student selects the number that could not be the next term in an arithmetic sequence (D).

Response Type: Multiple Choice, single correct response

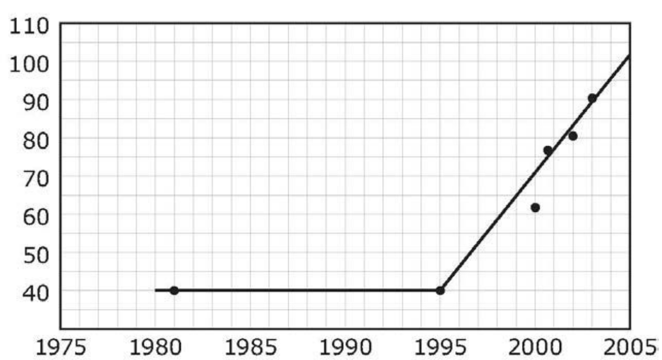
Claim 4 Modeling and Data Analysis Question Banks

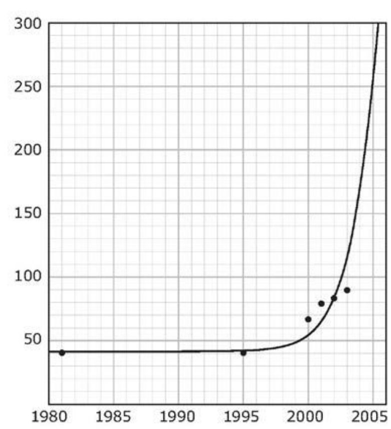
[Claim Descriptors and Targets](#)

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Example 1

Between 1980 and 1995, there was a single surviving group of Florida panthers that ranged from 30 to 50 individuals in number. In 1995, two females from a closely related species were introduced into this population, and the number of Florida panthers increased to 87 by 2003.
Part A:
Would you model this data with a piece-wise linear function or an exponential function? Select one of these models.
Either model is acceptable, but you must choose one. [piece-wise linear, exponential]
Part B:
You chose to model this data with a [auto-populates with student's choice]. This graph shows such a model.





[If the student chose a piece-wise linear function, they see the first graph. If they chose an exponential function, they see second graph.]

What would the approximate population of Florida panthers have been in 2005 according to the model you chose?

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Enter your answer in the response box.

Interaction: The student selects a model and then sees the corresponding graph. The student can change his/her choice.

Rubric: (1 point) Student selects piecewise linear or exponential, and then enters the appropriate estimate (between 98 and 100 if piecewise linear, and between 240 and 260 if exponential).

Response Type: Equation/Numeric

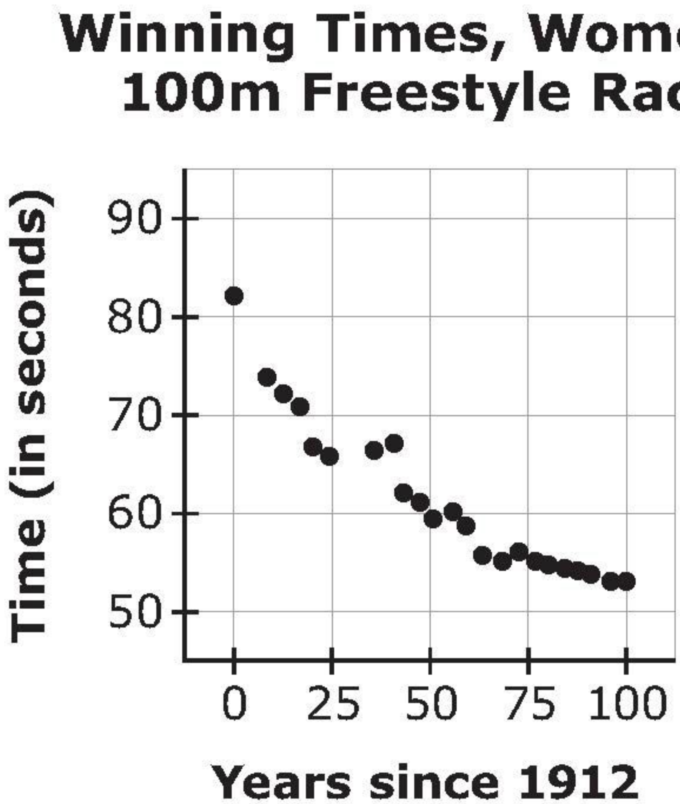
Example 2

freestyle race for the Olympic Games between 1912 and 2012.

Part A:
Let x be the year since 1912 and f(x) be the winning time in seconds. Enter either a linear or an exponential function that models the data in the response box.

Part B:
Greta Andersen from Denmark won the race in 1948 with a time of 66.3 seconds. What is the difference between your model's prediction and Greta's actual winning time? Enter your answer in the response box.

Part C:
What does your model predict the winning time will be in the 2016 Olympics? Enter your answer in the response box.



Rubric: (2 points) The student enters an expression for f(x) in the first response box such that |f(0) - 82| ≤ 10 and |f(100) - 53| ≤ 10. The student then enters ±(f(36)-66.3) within a reasonable tolerance in the second response box and enters f(104) in the third response box.
(1 point) The student enters an expression for f(x) in the first response box such that |f(0) - 82| ≤ 10 and |f(100) - 53| ≤ 10, or, the student enters an expression that does not meet these criteria but successfully enters either ±(f(36)-66.3) within a reasonable tolerance in the second response box or enters f(104) in the third response box (or both).

Response Type: Equation/Numeric - label each response type as follows: Part A: f(x) = , Part B:, and Part C:.

Data for the scatterplot for example item 4A.1c:

Year since 1912	0	8	12	16	20	24	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
Time	82.2	73.6	72.4	71	66.8	65.9	66.3	66.8	62	61.2	59.5	60	58.59	55.65	54.79	55.92	54.93	54.65	54.5	53.83	53.84	53.12	53

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