HS Target M

Domain, Target, Standards, DOK, Vertical Alignments, Achievement Levels, Evidence Required, Vocabulary, Response Types, Materials, Attributes, Question Types, and Question Banks (Examples)

Content Domain: Functions

Target M [m]: F-IF.C Analyze functions using different representations.

Standards included in Target M: F-IF.C, F-IF.C.7a, F-IF.C.7b, F-IF.C.7c, F-IF.C.7e, F-IF.C.8a, F-IF.C.8b, F-IF.C.9

F-IF.C Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions, such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Vertical Alignment

Related Grade 8 standards

8.F.A Define, evaluate, and compare functions.

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.B Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Achievement Level Descriptors

Level 1 Students should be able to graph a linear function by hand or by using technology. They should be able to compare properties of two linear functions represented in different ways. They should be able to identify equivalent forms of linear functions.

Level 2 Students should be able to graph linear and quadratic functions by hand; graph square root, cube root, piecewise-defined, polynomial, exponential, and logarithmic functions by hand or by using technology; compare properties of two quadratic or two other functions of the same type, i.e., linear to linear, represented in different ways; and understand equivalent forms of linear and quadratic functions. They should be able to compare properties of two trigonometric functions represented in the same way.

Level 3 Students should be able to analyze and compare properties of two functions of different types represented in different ways and understand equivalent forms of functions. They should be able to graph trigonometric functions by hand and by using technology.

Level 4 Students should be able to graph a variety of functions, including linear, quadratic, square root, cube root, piecewise-defined, polynomial, exponential, logarithmic, and trigonometric, by hand and by using technology. They should be able to analyze and explain relationships between various types of functions and the behaviors of the functions and be able to determine which equivalent form is most appropriate for a given task.

Evidence Required

1. Students graph functions expressed symbolically and show key features of the graph.

2. Retired this Evidence Required statement.

3. Students write an exponential function defined by an expression in an equivalent form using the properties of exponents to reveal and explain different properties of the function and to classify them as representing exponential growth or decay.

4. Students compare properties of two functions each represented in a different way (e.g., as equations, functions, tables, graphs, or written descriptions).

Vocabulary
quadratic, square root, cube root, piecewise-defined, polynomial, exponential, logarithmic, x-intercept, y-intercept, interval, relative maximum, relative minimum, symmetry, axis of symmetry, end behavior of a graph, limit, periodicity

Response Types
Graphing; Matching Table; Hot Spot; Multiple Choice, multiple correct; Equation/Numeric

Materials
linear functions, quadratic functions, square-root functions, cube-root functions, piecewise-defined functions, polynomial functions, exponential functions, logarithmic functions, table of values, description of a function, description of a property or key feature of a function

Attributes
Properties and key features include: x- and y-intercepts; intervals where the function is increasing, decreasing, positive, or negative; zeroes; relative maximums and minimums; symmetries; end behavior; and periodicity.

Claim 1: Concepts and Procedures (DOK 1, 2) Question Banks
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Claim 1 F-IF.C.7 DOK Level 2

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

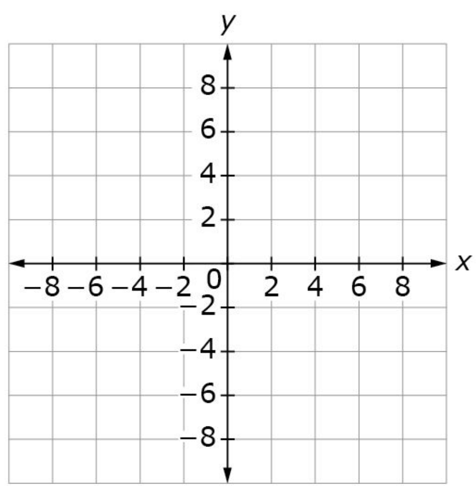
Evidence Required
Students graph functions expressed symbolically and show key features of the graph.

Question Type 1: The student is presented with a function and a coordinate grid.

1. Given a linear function with a slope of $\frac{2}{3}$ and a y-intercept of 2:
   - Using the Add Arrow tool, draw a line on the coordinate grid to graph the function.
   - Place a point on the line representing the x-intercept of the function.

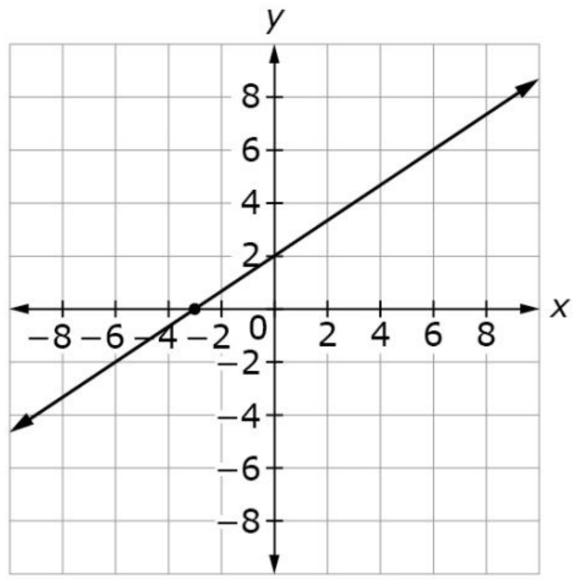
2. Given the function $y= \frac{2}{3}x+2$,
   - Using the Add Arrow tool, draw a line on the coordinate grid to graph the function.
   - Place a point on the line representing the x-intercept of the function.

3. Given the function $y= \frac{1}{2}|2x-1|+2$,
   - Use the Add Arrow tool to create a graph that represents the function.
   Response Type: Graphing
   - Place a point on the coordinate grid to show the y-intercept of the function.



Interaction: The student will graph lines using the Add Arrow tool and/or plot points using the Add Point tool.

Rubric: (2 points) The student graphs the correct line and plots the point at the correct location that represents a key feature [e.g., Example Stem 1, draws a correct line and plots the x-intercept located at (-3, 0)]. (1 point) The student graphs the correct line or plots the point at the correct location that represents a key feature [e.g., Example Stem 1, draws a correct line OR plots the x-intercept located at (-3, 0)].



Response Type: Graphing

Claim 1 F-IF.C.7 DOK Level 2

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Evidence Required
Students graph functions expressed symbolically and show key features of the graph.

Question Type 1: The student is presented with a function and a coordinate grid.

1. Given the function $y = -x^2 + x + 6$,
   - Place a point on the coordinate grid to show each x-intercept of the function.
   - Place a point on the coordinate grid to show the maximum value of the function.

2. Given the function $y = \sqrt{x+4} - 1$,
   - Place a point on the coordinate grid to show each x-intercept of the function
   - Place a point on the coordinate grid to show the y-intercept of the function.

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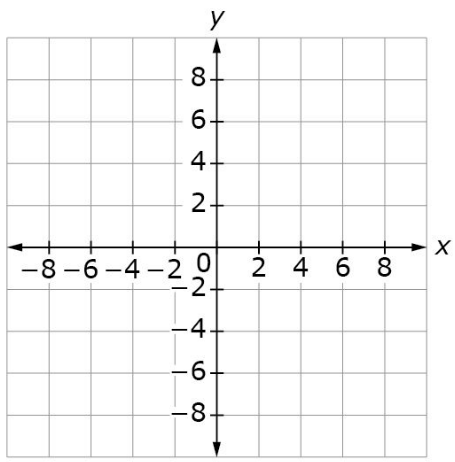
3. Given the function $y=^{3}\sqrt{x-1}+2$,

- Place a point on the coordinate grid to show the x-intercept of the function.
- Place a point on the coordinate grid to show the y-intercept of the function.

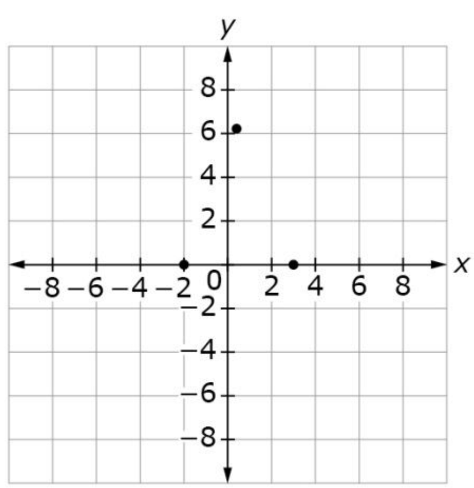
7. Given the function yy=8log(xx+4),

- Place a point on the coordinate grid to show the x-intercept of the function.
- Place a point on the coordinate grid to show the y-intercept of the function.



Interaction: The student will graph lines using the Add Arrow tool appropriate to the example stem (single or double) and/or plot points using the Add Point tool.

Rubric: (2 points) The student plots the correct points that represent each different key feature. (e.g., Example Stem 1, student plots both x-intercepts and the maximum value). (1 point) The student correctly plots 1 of 2 key features called (e.g., in Example Stem 1, student plots the maximum value only OR both x-intercepts only.)



Note: Both x-intercepts represent one key feature so both are required to earn a point. For example, in Example Stem 1 both x-intercepts represent one key feature (1 point), and the maximum point represents another key feature (1 point).

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Response Type: Graphing

Claim 1 F-IF.C.8a DOK Level 2

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function: a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Evidence Required
Students write a quadratic function defined by an expression in equivalent factored form and completing the square form to reveal zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context

Question Type 1: The student is presented with a quadratic function.

1. Enter an equation for the line of symmetry for the function defined by $f(x)=-8x^2+16x+2$.

Rubric: (1 point) The student enters the correct equation (e.g., x = 1).

Response Type: Equation/Numeric

Claim 1 F-IF.C.8b DOK Level 1

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function: b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions, such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

Evidence Required
Students write an exponential function defined by an expression in an equivalent form using the properties of exponents to reveal and explain different properties of the function and to classify them as representing exponential growth or decay.

Question Type 1: The student is presented with multiple functions.

1. Determine whether each function represents exponential growth or decay.

Select the correct option for each function.

Function	Growth	Decay
$f(x) = \left(\frac{1}{2}\right)^{x}$		
$f(x) = \left(\frac{3}{2}\right)^{4x}$		
$f(x) = \left(\frac{7}{8}\right)^{4x}$		
$f(x) = \left(\frac{4}{3}\right)^{\frac{x}{12}}$		
$f(x) = 3\left(\frac{1}{3}\right)^{\frac{x}{12}}$		

Rubric: (1 point) The student correctly sorts the exponential functions (e.g., Decay, Growth, Decay, Growth, Decay).

Response Type: Matching Table

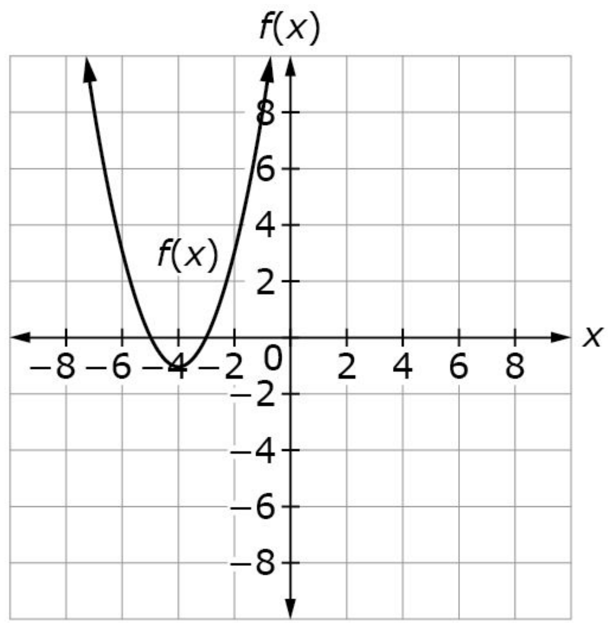
Claim 1 F-IF.C.9 DOK Level 2

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Evidence Required
Students compare properties of two functions represented in different ways (e.g., as equations, tables, graphs, or written descriptions).

Question Type 1: The student is presented with two functions that must be represented in two different ways. Functions can be represented as a table of values, a graph, a function equation, or a written description.

1. The graph represents f(x) and the table shows some values of another quadratic function g(x).



x	−4	−3	−2	−1	0	1	2	3	4	5	6
***g*(*x*)**	0	−9	−16	−21	−24	−25	−24	−21	−16	−9	0

Select whether each statement is True or False about the given functions.

Statement	True	False
The minimum value of $f(x)$ is greater than the minimum value of $g(x)$.		
The value of x when $f(x)$ is at its minimum is less than the value of x when $g(x)$ is at its minimum.		

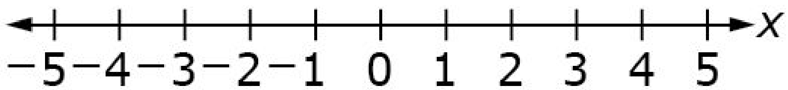
Rubric: (1 point) The student correctly identifies each statement as True or False (e.g., TT).

Response Type: Matching Table

Question Type 2: The student is given two different functions (square root, cube root, piecewise-defined, or absolute value) and a number line representing the x-axis, and asked to indicate where the functions have a shared key feature.

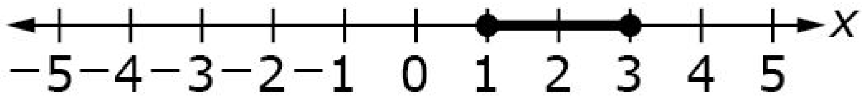
1. In which interval(s) on the x-axis are the functions $f(x)=\frac{1}{2}|2x|+2$ and $g(x)=-2x^2+12x-16$ increasing?

Click the interval(s) on the number line that represents where both functions are increasing.



Interaction: The student will click on intervals on the number line using Hot Spots.

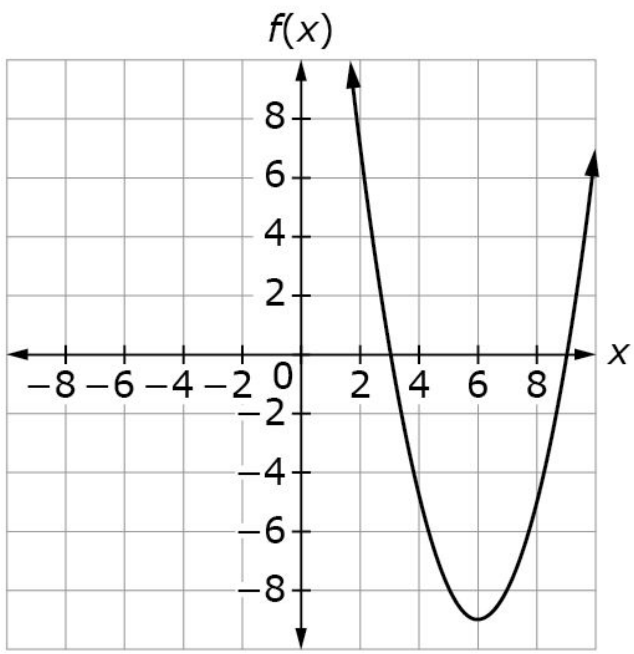
Rubric: (1 point) The student clicks on the correct intervals (e.g., [1, 3]).



Response Type: Hot Spot

Question Type 3: The student is presented with the graph of a quadratic function and a table of equations that may or may not represent the function.

1. Determine whether each equation in the table represents the graph of the function shown. Select Yes or No for each equation.



Function	Yes	No
$f(x) = (x-3)(x-9)$		
$f(x) = (x+3)(x-9)$		
$f(x) = (x+6)(x-9)$		
$f(x) = (x-3)^2 - 18$		
$f(x) = (x-6)^2 - 9$		

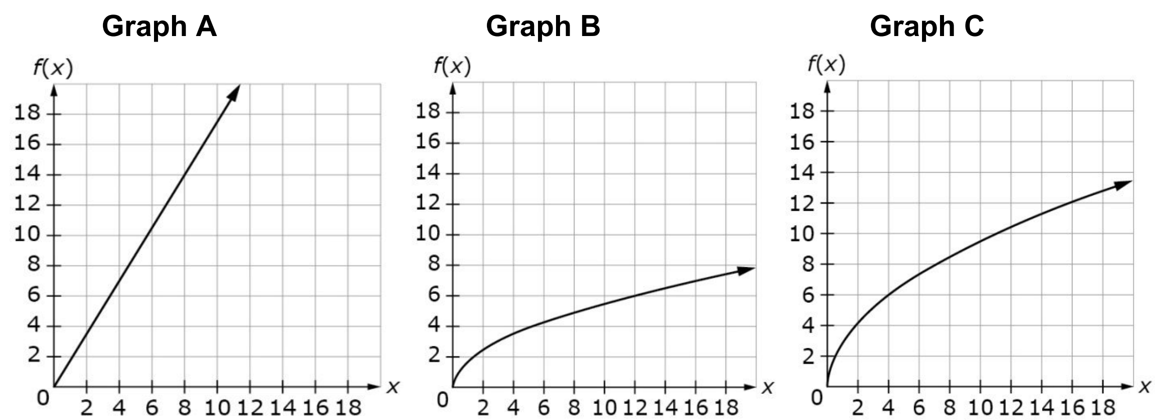
Rubric: (1 point) Student correctly selects the functions that could be represented by the given graph (e.g., YNNNY).

Response Type: Matching Table

Question Type 4: The student is presented with three functions in various forms (graphs, table of values, etc.) and a matching table that includes the equations of the three functions.

Note: If tables are given, the ordered pairs should show key features (zeros, etc.).

1. Select the appropriate box to indicate the match of each graph to its equation.



Equation	Graph A	Graph B	Graph C
$f(x) = x\sqrt{3}$			
$f(x) = 3\sqrt{x}$			
$f(x) = \sqrt{3x}$			

Rubric: (1 point) The student correctly matches the functions with the graph (e.g., Table A, Table C, Table B).

2. Select the appropriate box to indicate the match of each table of values to its equation.

Table A	
x	$f(x)$
1	1.73
2	3.46
4	6.92
6	10.38
8	13.84

Table B	
x	$f(x)$
1	1.73
2	2.45
4	3.46
6	4.24
8	4.90

Table C	
x	$f(x)$
1	3.00
2	4.24
4	6.00
6	7.35
8	8.49

Equation	Table A	Table B	Table C
$f(x) = x\sqrt{3}$			
$f(x) = 3\sqrt{x}$			
$f(x) = \sqrt{3x}$			

Rubric: (1 point) The student correctly matches the functions with the table (e.g., Table A, Table C, Table B).

Response Type: Matching Table

Claim 2 Problem Solving Questions Banks

Claim Descriptors and Targets

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Example 1

The function f models the amount of a chemical that can be extracted from a mixture given the percent of ethanol used in the extraction process, x, for $0 \leq xx \leq 100$. What value of x between 0 and 100 gives the maximum value for this function?

$$f(x) = -0.002x^3 + 0.255x^2 - 4.5x + 165$$

Enter your answer in the response box.

Interaction: The student uses the graphing calculator and estimates the value for x that produces the maximum value of the function.

Rubric: (1 point) The student enters the correct value for x, with a tolerance of 5 percentage points (e.g., 75+/-5)

Response Type: Equation/Numeric

Example 2

Lisa plans to build a rectangular fenced garden. She has 500 feet of fencing to use for the project. She determines that if her garden has a width of w meters, then the area of her garden A(w), in square meters, is given by the following function.

$$A(w) = 250w - w^2$$

Which of the following expressions is equivalent to $250w - w^2$?

A. $-(w - 125)^2$
B. $-(w - 250)^2$
C. $-(w - 125)^2 + 15{,}625$
D. $-(w - 250)^2 + 62{,}500$

What is the largest garden area, in square meters, that Lisa can enclose with 500 meters of fencing?

Enter your answer in the response box.

Area: ☐

Rubric: (2 points) The student selects the correct expression (C) and identifies the largest area possible (15625).
(1 point) The student is able to correctly identify the expression or identify the largest area.

Response Type: Multiple choice/single answer and Equation/numeric

Claim 3 Communicating Reasoning Question Banks
Claim Descriptors and Targets

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Example 1

Consider the piecewise-defined function given by
For how many real values of x does f(x)=0?

$$f(x) = \begin{cases} 2x - 7 & \text{for } x < -1 \\ x^2 + 3 & \text{for } -1 \leq x < 3 \\ x - 1 & \text{for } x \geq 3 \end{cases}$$

A. 0
B. 1
C. 2
D. 3

Rubric: (1 point) The student selects the correct number of zeros for the function (A).

Response Type: Multiple Choice, single correct response