HS Target L

Domain, Target, Standards, DOK, Vertical Alignments, Achievement Levels, Evidence Required, Vocabulary, Response Types, Materials, Attributes, Question Types, and Question Banks (Examples)

Content Domain: Functions

Target L [m]: F-IF.B Interpret functions that arise in applications in terms of the context.

Standards included in Target L: N-RN.A, N-RN.A.1, N-RN.A.

F-IF.B Interpret functions that arise in applications in terms of the context.

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Vertical Alignment

Related Grade 8 standards

8.F.A Define, evaluate, and compare functions.

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.B Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Achievement Level Descriptors

Level 1 Students should be able to interpret linear functions in context, and given the key features of a linear graph, they should be able to identify the appropriate graph.

Level 2 Students should be able to interpret quadratic and other polynomial functions in two variables in context of the situation, and given the key features of a graph of a polynomial function, they should be able to identify the appropriate graph. They should be able to specify the average rate of change from an equation of a linear function and approximate it from a graph of a linear function.

Level 3 Students should be able to graph various types of functions and interpret and relate key features, including range and domain, in familiar or scaffolded contexts. They should be able to specify the average rate of change of a function on a given domain from its equation or approximate the average rate of change of a function from its graph.

Level 4 Students should be able to interpret complex key features such as holes, symmetries, and end behavior of graphs and functions in unfamiliar problems or contexts.

Evidence Required

1. The student interprets key features of a graph or a table representing a function modeling a relationship between two quantities.

2. The student sketches graphs showing key features given a verbal description of a relationship between two quantities that can be modeled with a function.

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3. The student relates the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

4. The student calculates and interprets the average rate of change of a function presented symbolically or as a table and estimates the rate of change of a function from a graph.

Vocabulary
x-intercept, y-intercept, interval, increasing interval, decreasing interval, relative maximum, relative minimum, symmetry, axis of symmetry, end behavior of a graph, limit, periodicity, average rate of change

Response Types
Matching Table; Multiple Choice, single correct response; Multiple Choice, multiple correct response; Equation/Numeric; Hot Spot; Graphing

Materials
written description of key features of a function, graphs of functions in the coordinate plane, tables containing domain and range values of functions, functions presented symbolically

Attributes
Key features include x- and y-intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
Functions are in two variables.

Claim 1: Concepts and Procedures (DOK 1, 2) Question Banks
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

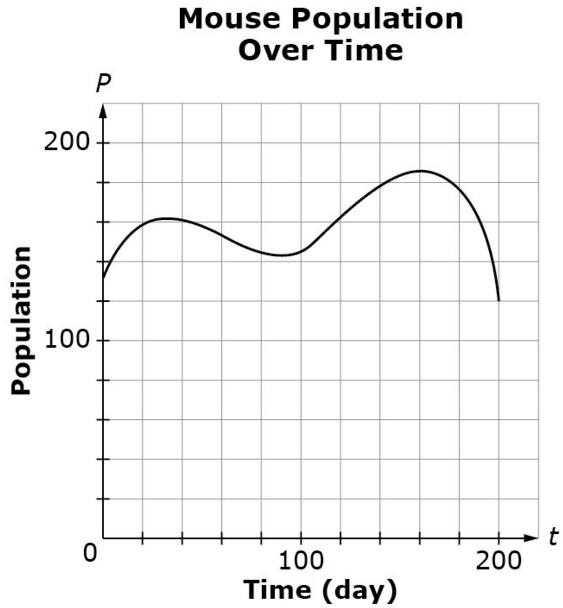
Claim 1 F-IF.B.4 DOK Level 1

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Evidence Required
The student interprets key features of a graph or a table representing a function modeling a relationship between two quantities.

Question Type 1: The student is given a graph representing a function that models the relationship between two quantities in a real-world situation familiar to 15- to 17-year-olds, e.g., temperature change over time, or population change over a period of time.

1. This graph shows the population of mice in a study, modeled as a function of time. The study begins on day 0 and ends on day 200.



Determine whether each statement is true according to the graph. Select True or False for each statement.

Statement	True	False
The mouse population was decreasing between day 40 and day 80.		
The least number of mice during the study was 130.		
The mouse population was at its greatest around day 160.		
There are two intervals of time where the mouse population is decreasing.		

Rubric: (1 point) The student correctly selects true or false for each of the interpretations of key features of the graph (e.g. T, F, T, T).

Response Type: Matching Table

Question Type 2: The student is given a graph representing a function that models the relationship between two quantities in a real-world situation familiar to 15- to 17-year-olds, e.g., temperature change over time, or population change over a period of time.

1. A bird flies out of its nest. This graph represents the distance it flies from its nest (in meters) as a function of time (in seconds).

Drag the star to mark the point on the graph that represents the bird's greatest distance from its nest. Then drag the circle to mark the point that represents the bird's return to its nest.
Interaction: The student drags the star and circle to the correct points on the graph.

Rubric: (1 point) The student correctly identifies the point representing the bird's farthest distance from the nest and the point where the bird returns [e.g., approximately (7, 5.2) and (8.7, 0)].

Response Type: Hot Spot

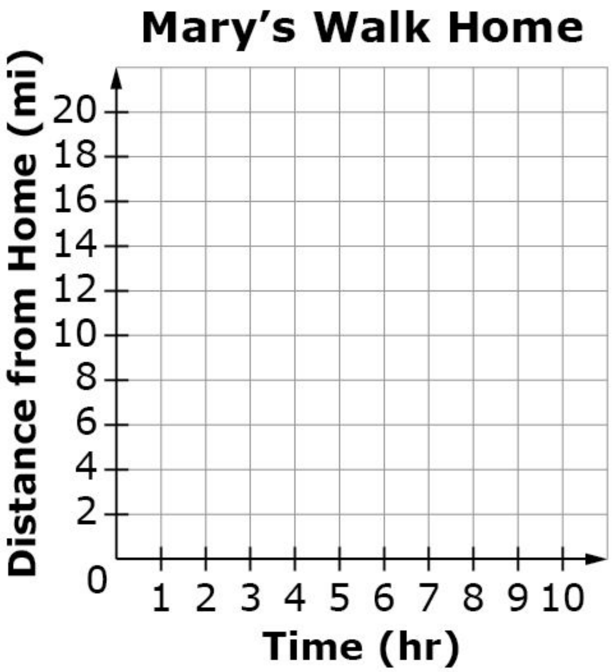
Claim 1 F-IF.B.4 DOK Level 2

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Evidence Required
The student sketches graphs showing key features given a verbal description of a relationship between two quantities that can be modeled with a function.

Question Type 1: The student is presented with a contextual situation, familiar to 15- to 17-year-olds, where a function can model a relationship between two quantities.

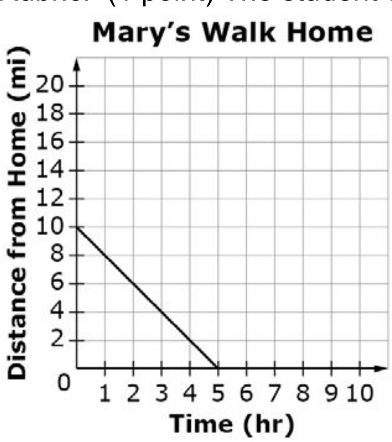
1. Mary is 10 miles from her home.
   - She is returning home, walking at a constant speed of 2 miles per hour.
   - Her distance from home can be modeled as a function of time.



Use the Add Point and Connect Line tools to graph Mary's distance from home as a function of time.

Interaction: The student uses the Add Point tool to place points on the grid, and the Connect Line tool to connect the points.

Rubric: (1 point) The student creates the graph correctly (e.g., see below).



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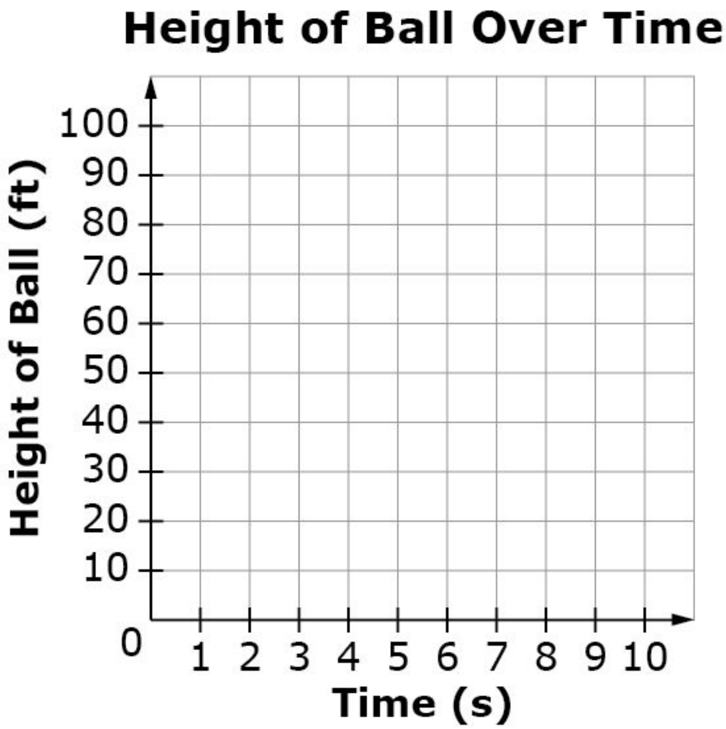
2. A ball is on the ground. Jon kicks the ball into the air at s = 0. Assume that the height of the ball can be modeled as a quadratic function with respect to time. It reaches a maximum height of 64 feet and lands on the ground 4 seconds later.

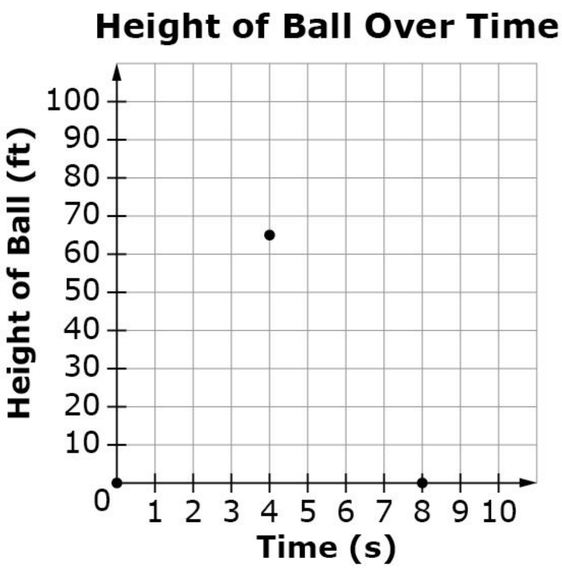


Use the Add Point tool to plot the points on the grid that represent

- when John kicks the ball,
- the ball at its highest point, and
- when the ball lands on the ground.

Interaction: The student uses the Add Point tool to place points on the grid.

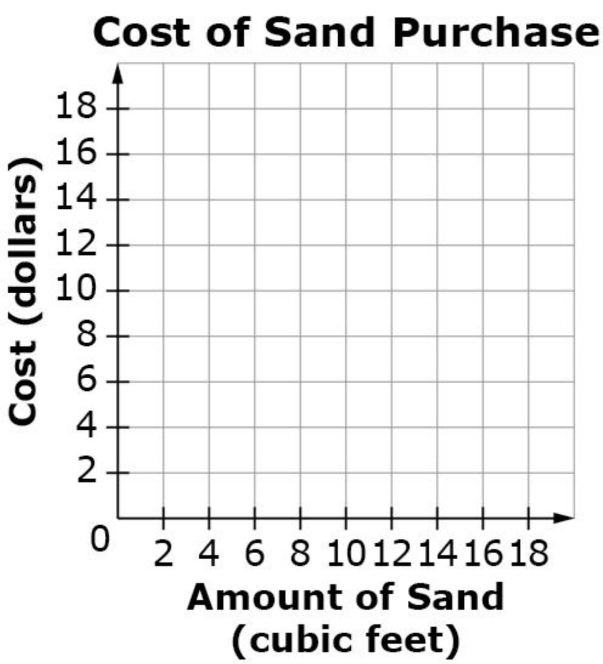
Rubric: (1 point) The student plots the points correctly (e.g., see below).



Example Stem 3: A company is building a playground and needs to buy sand. The cost of sand is a function of the amount of sand purchased.

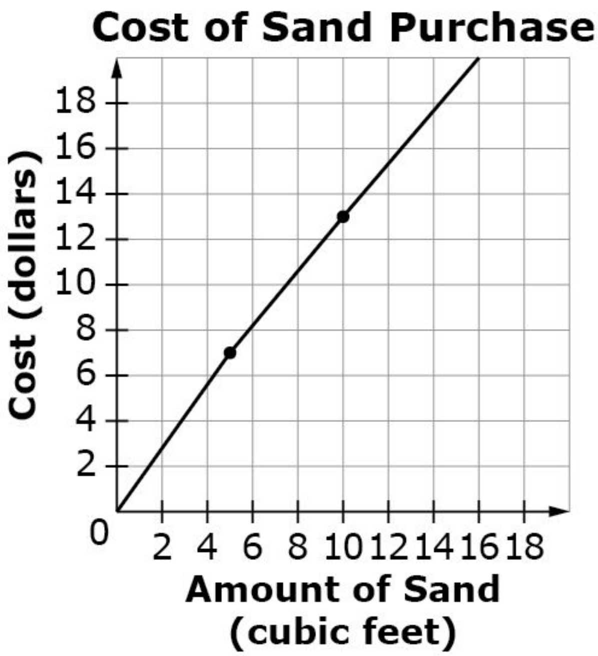
- The first 5 cubic feet cost $1.50 per cubic foot.
- An amount greater than 5 cubic feet and less than or equal to 10 cubic feet costs $1.25 per cubic foot.
- An amount over 10 cubic feet costs $1.00 per cubic foot.

Use the Add Point and Connect Line tools to create a graph to show the total cost of the sand (in dollars) as a function of the amount of sand purchased (in cubic feet).



Interaction: The student uses the Add Point tool and Connect Line tool to graph the linear segments of a piecewise function on the grid.

Rubric: (1 point) The student creates the graph correctly (e.g. see below).



Response Type: Graphing

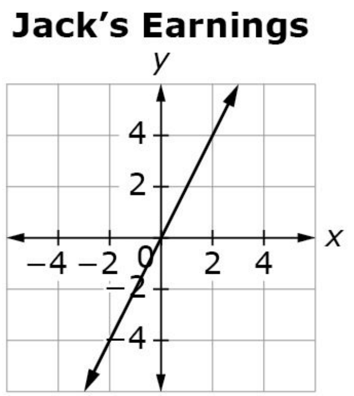
Claim 1 F-IF.B.5 DOK Level 2

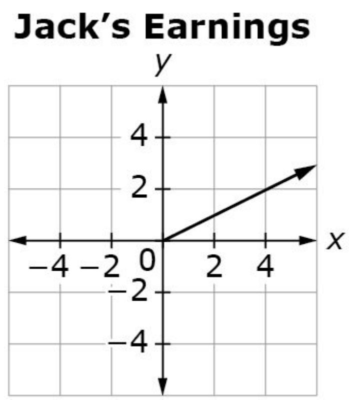
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

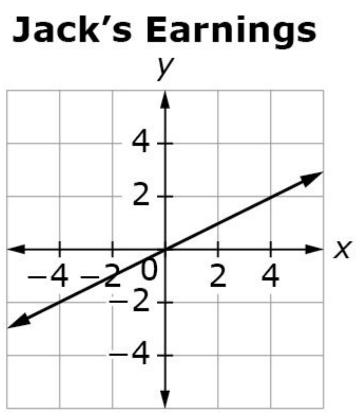
Evidence Required
The student relates the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

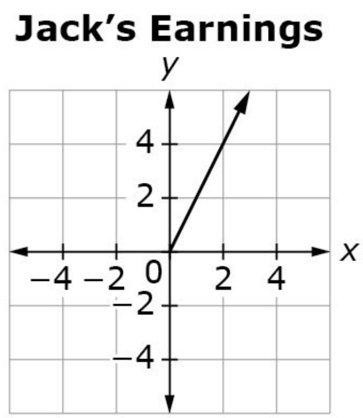
Question Type 1: The student is presented with four graphs of a function in the coordinate plane, with the graphs in various intervals of positive and negative x-values.

1. Select the graph that correctly represents the amount of money, y, Jack earns doing chores for x hours at $2 per hour.

A. 

B. 

C. 

D. 

Rubric: (1 point) The student identifies the correct graph (e.g., D).

Response Type: Multiple Choice, single correct response

Question Type 2: The student is presented with a contextual situation, and asked to identify the domain of the function modeled by the given situation.

1. Billy buys light bulbs in packs of 8 for $20. The shipping cost is $10 regardless of the number of packs bought. Billy has only $120 to spend.

If n is the number of packs of lightbulbs bought, then the cost per lightbulb, C, can be modeled as a function of n. Select the statement that correctly describes the domain of the function.

A. The domain is the set of all real numbers $1 \le n \le 6$.
B. The domain is the set of all real numbers $1 \le n \le 5$.
C. The domain is the set of all integers $1 \le n \le 6$.
D. The domain is the set of all integers $1 \le n \le 5$.

Rubric: (1 point) The student correctly selects the statement describing the domain or range of the function (e.g., D).

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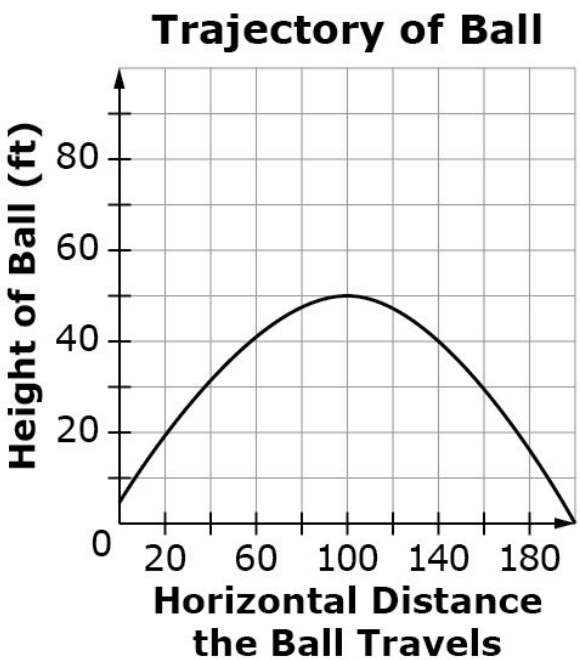
Response Types: Multiple Choice, single correct response

Question Type 3: The student is presented with a description and graph of contextual situation, and asked to identify all values that are in the domain of the function modeled by the given situation.

1. Sue hits a ball from a height of 4 feet. The height of the ball above the ground is a function of the horizontal distance the ball travels until it comes to rest on the ground. Consider this complete graph of the function.



Select all values that are in the domain of the function as shown in the graph.

A. –5 feet
B. 0 feet
C. 60 feet
D. 220 feet

Rubric: (1 point) The student correctly selects the values that are within the domain (e.g., B, C).

Response Types: Multiple Choice, multiple correct response

Question Type 4: The student is presented with an equation and several descriptions of contextual situations, and asked to identify appropriate values of the domain of the function modeled by the given situations.

1.Consider the function f(x) = 10x + 25. Identify an appropriate domain for the function if it is used to model each of the following contexts.

	An interval of real numbers	A subset of the integers
The total value *f*(*x*) in cents of a handful of *x* dimes and 1 quarter.		
The amount of water *f*(*x*) in a tank that starts with 25 gallons of water and is being filled by a hose at 10 gallons per hour after *x* hours.		
The total money raised *f*(*x*) if *x* people donate $10 each and one person donates $25.		

Rubric: (1 point) The student correctly identifies the appropriate kind of domain (integers, reals, integers).

Response Types: Matching Table

Claim 1 F-IF.B.6 DOK Level 2

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Evidence Required
The student calculates and interprets the average rate of change of a function presented symbolically or as a table and estimates the rate of change of a function from a graph.

Question Type 1: The student is presented with a function in symbolic form, representing a context familiar to 15- to 17-year-olds.

1. Craig records the number of minutes, m, it takes him to mow n lawns in a table.

n	1	2	3	4	5	6
m(*n*)	33	64	89	109	124	139

Select the average amount of time per lawn it takes Craig to mow the first 4 lawns. Round to the nearest minute per lawn.

A. 25 minutes per lawn
B. 27 minutes per lawn
C. 33 minutes per lawn
D. 74 minutes per lawn

Rubric: (1 point) The student identifies the correct value for the average rate of change (e.g., B).

Response Types: Multiple Choice, single correct response

Question Type 2: The student is presented with a nonlinear function in symbolic form.

1. During the first years of growth the height of a tree can be modeled with the function $h=-tt2+12tt+10$, where t is the time in years since being planted and h is the height in inches. Enter the average rate of change, in inches per year, from year 1 to year 5.

Rubric: (1 point) The student enters the correct answer for the average rate of change given the units (e.g., 6).

Response Type: Equation/Numeric

Claim 2 Problem Solving Questions Banks

Claim Descriptors and Targets

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Example 1

The height, in meters, of a golf ball t seconds after it was hit is given by the function h(t) = -9.8(t - 8)2 + 36. The number 36 appears in the expression that defines the function. What does it tell you about the golf ball?

A. The time it takes for the golf ball to hit the ground.
B. The time it takes for the golf ball to reach it's greatest height.
C. The greatest height the golf ball reaches.
D. The speed at which the golf ball is traveling.

Rubric: The student selects the appropriate answer choice (C).

Response Types: Multiple Choice, single correct response

Example 2

The relationship between the Fahrenheit (F) and Kelvin (K) scales for measuring temperatures can be represented by a linear function. A temperature of 68° F corresponds to 293.15° K, and a temperature of 185° F corresponds to 358.15° K.

Which statement is the best interpretation of the slope of the graph of this function?

A. A temperature of 0° K corresponds to −459.67° F.

B. A temperature of 0° F corresponds to 255.37° K.
C. For each change of one degree in the Fahrenheit scale, there is a change of 59 degree in the Kelvin scale.
D. For each change of one degree in the Fahrenheit scale, there is a change of 95 degree in the Kelvin scale.

Rubric: The student selects the appropriate answer choice (C).

Response Types: Multiple Choice, single correct response.

Claim 3 Communicating Reasoning Question Banks

Claim Descriptors and Targets

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Example 1

A student examines two graphs representing the functions $ff(xx)=xx+5$ and $gg(xx)=xx2+5$. The student notices that the graphs both have a y-intercept at the point (0, 5). The student makes the following claim:

“For any real number cc, the y-intercepts for the graphs of $yy=cc\ ff(xx)$ and $yy=cc\ gg(xx)$ are the same.”

Is this true or false?

If it is true, enter the y-coordinate of the y-intercept in terms of c. (0, [ ])

If it is false, enter the y-coordinates of the y-intercepts of the two graphs that are a counter-example. (0, [ ]); (0, [ ])