The Flexibility Formula

Facilitation Guide

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IOWA™ | Department of Education

State of Iowa
Department of Education
Grimes State Office Building
400 E. 14th Street
Des Moines, IA 50319-0146

Administration

McKenzie Snow, Director

PK – 12 Learning Division

Tina Wahlert, Division Administrator

Academics and Learner Supports

Tom Wood, Chief

Stefanie Wager, Administrative Consultant

Christi Donald, Mathematics Consultant

April Pforts, Mathematics Consultant

Melissa Carlson, Mathematics Consultant

In Collaboration with

Christina Tondevold, Build Math Minds

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What is the Flexibility Formula?

The Flexibility Formula is a four-part framework for building students' flexibility with numbers: **Understanding, Observing, Connecting, and Experiencing**. It rests on two main ideas:

1. Flexibility with numbers can't be directly taught; it's caught.
2. We can't implement what we don't understand.

Children catch math flexibility through the experiences they have and the connections they make. However, we can't implement those ideas if we don't understand what it means to have flexibility with numbers.

The four parts in The Flexibility Formula describe how a teacher creates the conditions for building students' math flexibility: building their own understanding of the eight number sense concepts, learning to observe what students actually understand, seeing how the concepts connect and to the rest of the math their students do, and then designing the experiences that let flexibility take hold.

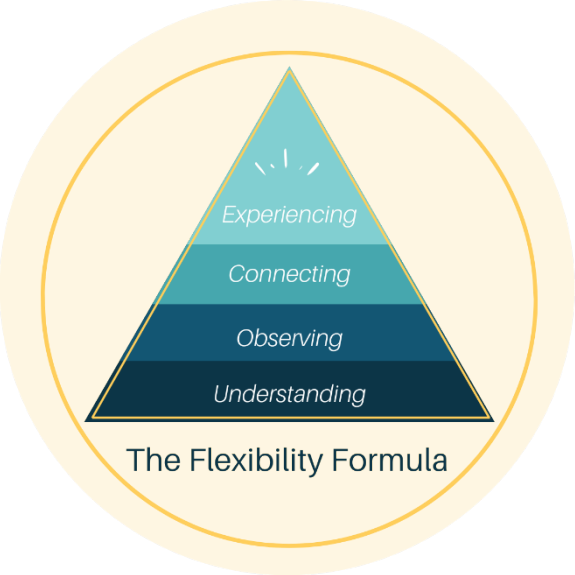


Figure 1: The Flexibility Formula

This guide is built on the assumption that you, as an instructional coach and/or leader, have already completed the modules in the course and focuses on what comes next: taking what you now understand and helping the teachers you coach build it for themselves, so that it lives in their classrooms, not just in your notes.

This guide was created in collaboration with Christina Tondevold. Christina is a nationally recognized mathematics educator who founded Build Math Minds and created The Flexibility Formula. Christina shares her own experiences and growth in number sense as she helps other educators grow their number sense.

Why It Matters

Educators bring varied experiences with mathematics instruction. This professional learning is designed to deepen understanding of number sense and support instructional practices that develop conceptual understanding, procedural fluency, and application.

The Research Behind It

The framework draws on four main research ideas:

- **Clements and Sarama's learning trajectories**, which map how young children develop the four early numeracy concepts – subitizing, verbal counting, object counting, and cardinality – before and beyond kindergarten.
- **Van de Walle et al.'s work on number relationships** (spatial relationships, one/two more and less, benchmarks, and part-part-whole) that students build around numbers 0-20 and how those then extend to all numbers.

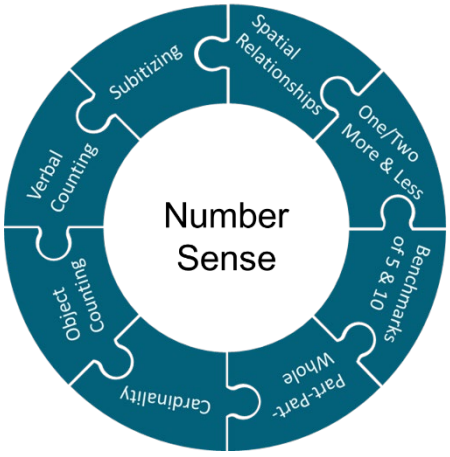


Figure 2: Number Sense Concepts

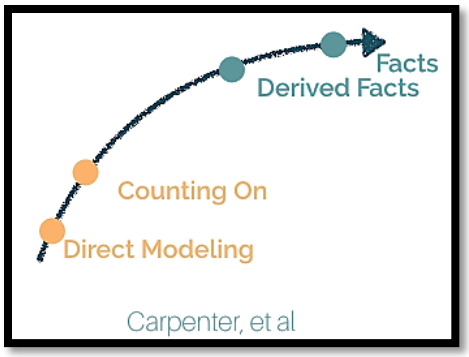


Figure 3: Number Sense Progression

- **Carpenter et al.'s work on Cognitively Guided Instruction (CGI)**, which shows how children's own thinking progresses from direct modeling to counting strategies to derived facts, and how that thinking should drive instruction.
- **Baroody and Dowker's research** on how students build arithmetic understanding and the flexible expertise to use it.

ESSA-Aligned Research

The Flexibility Formula incorporates instructional practices that are consistent with the recommendations in the What Works Clearinghouse (WWC) and Institute of Education Sciences (IES) practice guides and other cited research. The two IES/WWC practice guides listed below are especially relevant.

Teaching Math to Young Children (2013) makes five recommendations, several of which the Flexibility Formula addresses: teach number and operations using a developmental progression (eight number sense concepts as a progression rather than a checklist), use progress monitoring so instruction builds on what each child already knows (Module 2: Observing), teach children to view and describe their world mathematically (contextual problems), and dedicate time each day to mathematics and embed it in classroom routines (number sense routines). The research of Arthur Baroody and Douglas Clements, cited in the course research, is referenced a lot in this Practice Guide from the WWC & IES.

Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades (2021), another Practice Guide from the WWC & IES, adds more recommendations that are also things promoted inside The Flexibility Formula: using concrete and semi-concrete visual representations - drawings, arrays, number lines, strip diagrams - to help students organize mathematical ideas (concrete-representational-abstract progression), instruction on word problems built around common underlying structures (contextual problems), and brief, regular practice building fluent retrieval of basic facts (practice as one of the three experiences).

So, when a teacher asks "is there evidence for this?", the answer isn't just "Christina said so in the course" - it's that the moves you're asking them to make are the same ones the research evidence base recommends.

How to Access The Flexibility Formula Professional Learning Modules

Registration for The Flexibility Formula is open all year. Below are the steps for how to register and access the professional learning modules.

1. Please ensure the following email addresses are whitelisted in your district to minimize technology-related issues: info@iowa-buildmathminds.com and info@buildmathminds.com. You may need to contact your district's IT department. If you skip over this step, it may cause a hard bounce back that could disrupt future correspondence, send emails to your spam folder, or make the website inaccessible on your work device.
2. Register for The Flexibility Formula in Build Math Minds through the Iowa Department of Education.

3. You will receive an email with directions, including a password for registering in the Build Math Minds platform.
4. Once you register in the Build Math Minds platform, you will receive an email to verify your registration. If you do not receive the email, then you will need to check your spam folder and search for an email from info@iowa-buildmathminds.com.
5. After verification, you will have access to the following courses: The Flexibility Formula PreK-2, The Flexibility Formula 3-5, and The Flexibility Formula: Coaches Edition. Your page should look similar to the image below once you log in. If you are a 6th grade teacher, please use The Flexibility Formula 3-5 course as the content is appropriate and standards-aligned for grade 6.

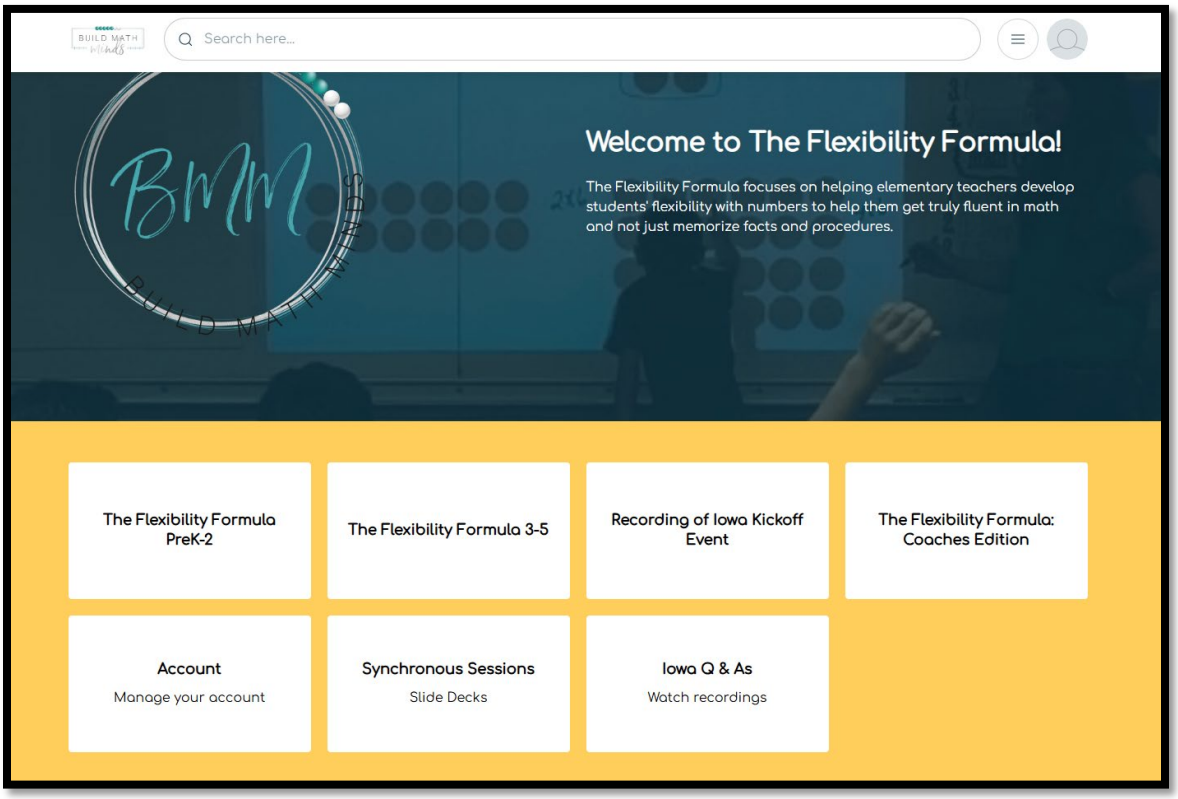


Figure 4: Build Math Minds Platform

How to Engage with This Facilitation Guide: The Coaching Routine

The most effective way to bring The Flexibility Formula into classrooms isn't to do hours of teachers sitting and listening to professional development from you - it's to move them through a cycle that lets them catch it the same way their students will. The Coaching Routine has five steps: **Immersion, Deconstruction, Connection, Application, and Reflection**.

That coaching routine is the vehicle. Think of the eight number sense concepts as the cargo. In any given cycle you choose a concept - like Benchmark Numbers - and run it through all five pieces: teachers experience a Benchmark task as learners (Immersion), unpack how it builds that specific concept (Deconstruction), learn more information about Benchmark tasks and link it to where Benchmarks show up across their curriculum (Connection), design and teach their own version (Application), and debrief what their students' thinking revealed (Reflection).

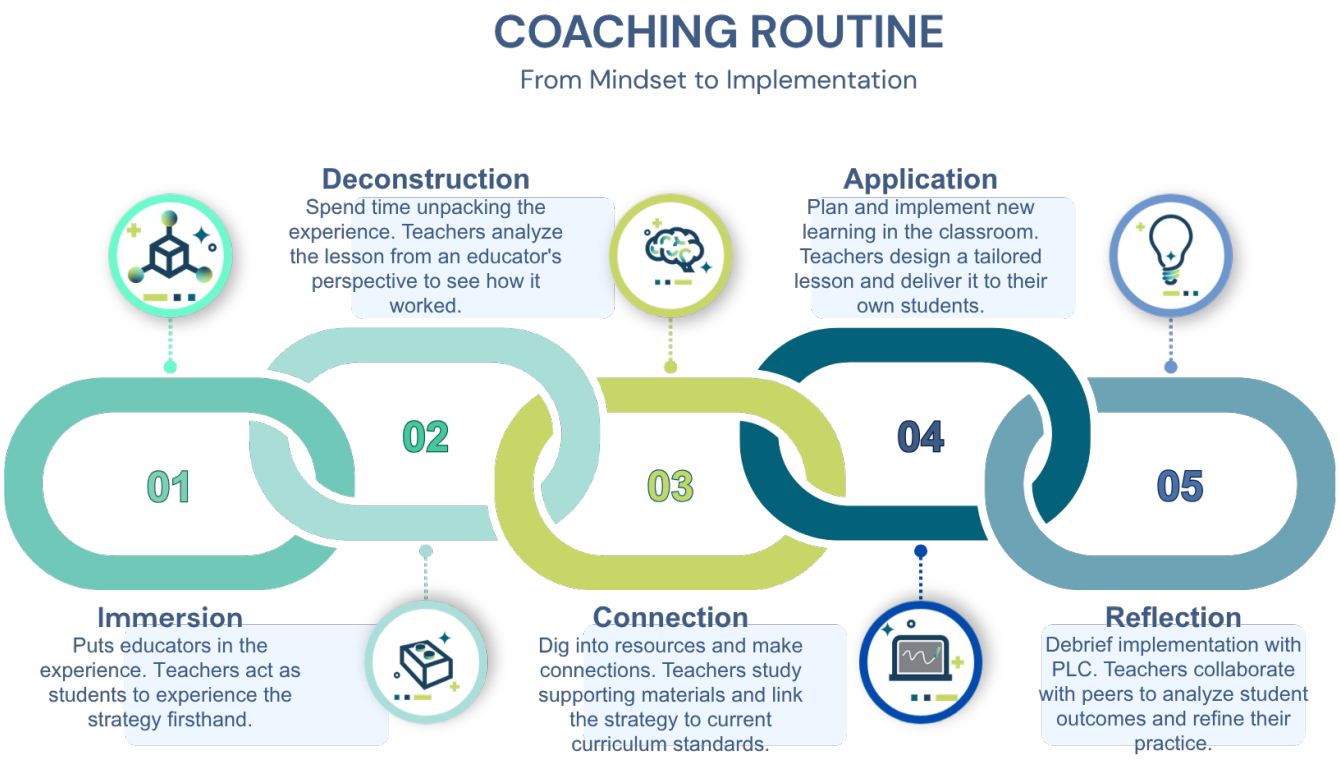


Figure 5: Coaching Routine

As you move from the learner of the content in The Flexibility Formula to the facilitator of the learning for your staff, this guide provides ideas for every part of the coaching routine. For each module, you'll find Immersion, Deconstruction, Connection, Application, and Reflection options to use with your staff. Educators will engage in the online modules during the Connection portion of the Coaching Routine. This guide provides a quick description and length of each video. The Iowa Department of Education's The Flexibility Formula Individual Reflection Workbook is a companion document for educators to use while watching the videos to assist with making connections and includes the application options. The translation work is already done, so you can spend your energy where it counts: alongside your teachers. Use them to Build the Math Minds of your teachers so that they can Build the Math Minds of their students.

Module 0: Welcome & Introduction

Overview

The Flexibility Formula course, led by Christina Tondevold, enhances both teaching flexibility in mathematics and students' number flexibility. It consists of four key parts: understanding math concepts, observing student thinking, connecting number sense to broader math ideas, and experiencing practical applications and the types of experiences students should have in mathematics. This entirely online course requires internet access for video viewing and includes a walkthrough for navigation.

Immersion Experience

To start your work with educators, one of the biggest mindset shifts you may need to help them make is that math ***can*** and ***should*** be done in multiple ways. These ways are dependent upon the learner who is solving the problem, along with the type of problem ***and*** the numbers in the problem. Not all problems are solved the same way, and not all learners solve the same problem the same way.

We want you to provide an experience that opens teachers' minds to multiple ways to solve problems, and this also gives you an opportunity to help educators see connections between 'formal' and 'informal' ways of solving problems. Often, as teachers, we think the informal ways are inefficient and bad ways to work out problems, when really those might just be some of the coolest ways to get answers.

By helping educators see that, rather than simply telling them, they will then be more open to allowing students to solve problems in ways that make sense to them.

Approximate Time: 45 minutes – 1 hour

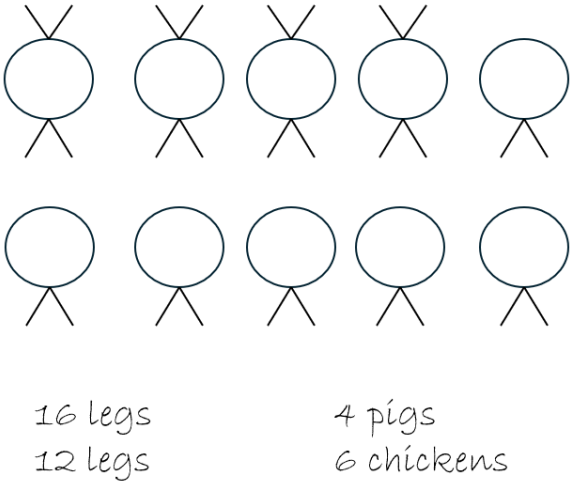
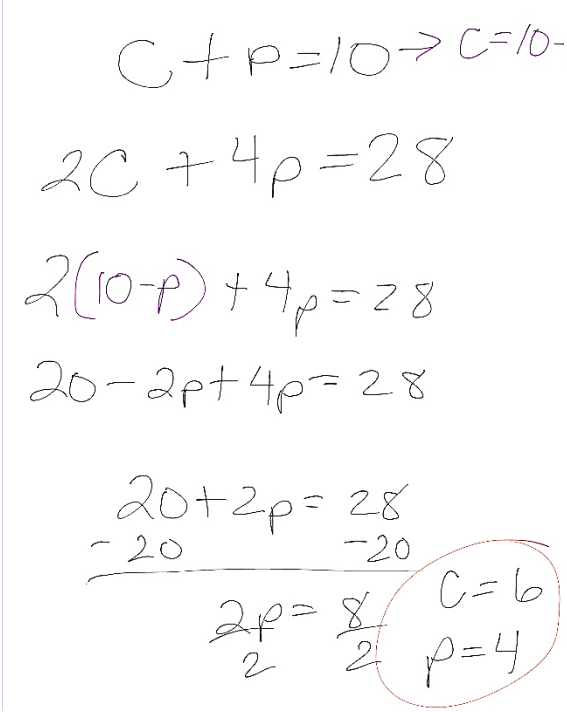
Professional Learning Objective: Participants will understand the power of multiple ways to solve mathematical problems.

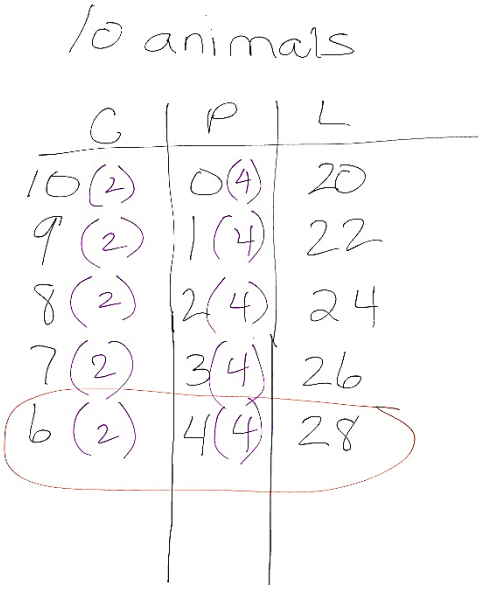
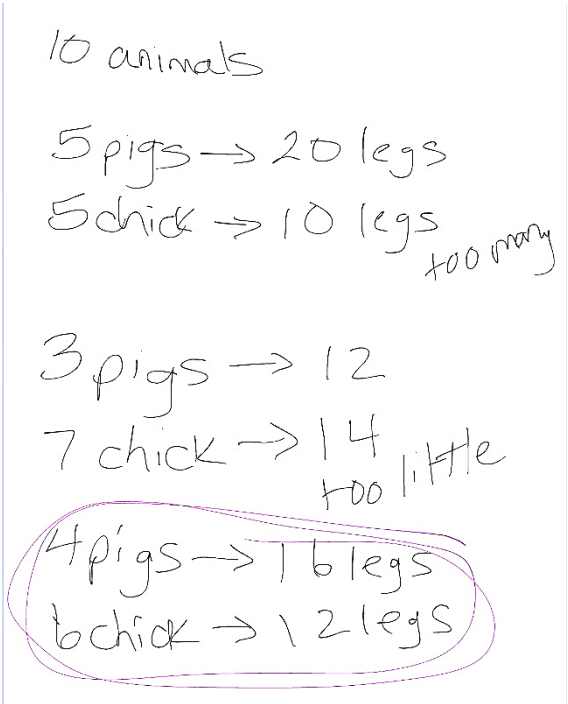
Resources Needed:

- The Farm Problem or The Café Problem (or any similar problem…there are a couple of versions, so you can pick the one you like or have different ones in case you use it in more than one situation)
- Whiteboard or vertical surfaces for educators to share their strategies

Facilitation Instructions

Segment	Instructions
Preparation	The setup of this experience gives you a great opportunity to do the 5 Practices by Smith & Stein as you facilitate: 1. **Anticipate** the different ways students (or your teachers) could solve a problem and the strategies they might use. We have crafted the typical responses for you so the anticipating practice is already done. 2. **Monitor** to see what strategies and approaches are being used as the students (or educators) are working through the problem. 3. **Select** which solution strategies are valuable to discuss as a whole group. 4. **Sequence** the presentation of solution strategies so that it is in a logical order, not just "who wants to share next?" 5. **Connect** the different solution strategies and the big mathematical ideas within them.

Segment	Instructions
Anticipate	**Starting with the Minimum Strategy** – This drawing is actually a sophisticated mathematical strategy where you start with the minimum amount (by putting two legs on each 'animal' because even the pigs have 2 legs), and then work up to the exact amount given in the problem.  *Figure 6: Visual Solution Method*
Anticipate	**Algebraic Systems of Equations Strategy** – This system of equations can be solved differently, so you may see the actual solving of it not look like this, but the general idea of taking the two constraints and writing them as equations would be the same.  *Figure 7: System of Equations Method*

Segment	Instructions
Anticipate	**Organized Chart Strategy** – With the small numbers in this problem, the need for an organized chart to keep track of 'guesses' isn't really necessary, and you may not have anyone who actually does it. You might have to be the person sharing this strategy. It's an important one to share so that you can help participants see patterns and connections.  10 animals C | P | L 10 (2) | 0 (4) | 20 9 (2) | 1 (4) | 22 8 (2) | 2 (4) | 24 7 (2) | 3 (4) | 26 6 (2) | 4 (4) | 28 *Figure 8: Table Solution Method*
Anticipate	**Guess & Check Strategy** – This tends to be the most popular way to solve these types of problems, even for adults. However, if the quantities were larger, it wouldn't be as easy to eventually get to the answer.  10 animals 5 pigs → 20 legs 5 chick → 10 legs too many 3 pigs → 12 7 chick → 14 too little 4 pigs → 16 legs 6 chick → 12 legs *Figure 9: Guess and Check Method*

Segment	Instructions
Monitor	If no one comes up with the algebraic way or no one has even thought to use algebra, bring it up: "As I'm walking around, I don't see anyone using algebra to solve this problem. If you've already found a way to solve it, I want to challenge you to see if you can remember that algebra from back in high school that we haven't used since to solve this problem." Do the same for the picture: "If you are struggling with where to even start with this problem, I've given this problem to 1st graders…how do you think they approached the problem? Try getting out of your head and try to solve it like a 1st grader."
Select	Select one person who did each of the four solution strategies listed above in Anticipate. Even if they didn't finish the problem, have them share how they have started/attempted to solve it, and then as a group you can all help finish it. **Sequencing:** 1. Starting with the minimum 2. Guess & Check (typically the most common strategy) 3. Organized Chart 4. Systems of Equations This sequence starts with the more informal solution strategies and progresses to the more formal strategies.
Connect	This part should not be skipped. It is essential to help the educators see the connections between strategies. Here are some possible questions you can ask to help participants see connections if they do not bring them up: • *Talk in your group about the similarities you see among strategies.* (Then have them share those with the group to hopefully have a discussion about how they are all connected.) • *Do you see the system of equations in any of these other strategies?* (**Hint**: It appears in all of them.) • *In the chart, what patterns do you notice with the numbers? Why do those appear?* • *In the chart, the two equations are inside the chart. Talk at your tables to discuss where the C + P = 10 and 2C + 4P = 28 can be seen.*

Deconstruction

Often, as teachers, we think informal ways are inefficient and bad ways to work out problems, when really those might just be some of the coolest ways to get answers.

By helping educators see that, rather than simply telling them, they will then be more open to allowing students to solve problems in ways that make sense to them. Your closing conversation should help them relate the ways they felt solving this problem to how children feel in the classroom. Here are some sample questions you could use to prompt the closing discussion:

- *What if I had told all of you to solve this problem using systems of equations only, how would you have felt?*
- *How would that have changed this lesson and what you took away from it?*
- *What teaching strategies were used in this experience?*

- *How did these strategies support multiple ways to solve the problem?*

Connection

The connection phase of this module is where educators view the videos and individually reflect by recording their thoughts in The Flexibility Formula Individual Reflection Workbook. Educators should be making connections from the experience at the beginning of the module to content in the videos and reading.

Videos in the Module

Video	Description	Length
Welcome	This video provides an overview of the course and Module 0.	15:25
Why Focus on Flexibility?	This video explains why flexibility is essential for developing true mathematical fluency in students, which goes beyond simply arriving at a correct answer.	18:59
Understanding & Observing	This video provides a short overview of modules 1 and 2 in the course.	4:50
Connecting & Experiencing	This video provides a short overview of modules 3 and 4 in the course.	5:40
Total		**44 minutes**

Resources in the Module

- *Developing Computational Fluency with Whole Numbers in the Elementary Grades* by Susan J. Russell
- Math Flips by Berkeley Everett

Application

The Iowa Department of Education has defined fluency as “the ability to apply procedures efficiently, flexibly, and accurately, including fact, computational, and procedural fluency” (Iowa Department of Education, 2024). Kindergarten through grade six have standards that include end-of-grade-level fluency expectations. This module’s application section (located in The Flexibility Formula Individual Reflection Workbook) asks educators to dig into their grade-level fluency standards and expectations and make connections with what they will be learning in The Flexibility Formula.

Reflection in PLCs

Facilitate PLCs through a discussion and reflection on their learning from Module 0. Here are some possible prompts/questions to get the conversation started.

- Unpack the grade-level fluency standards using the definition for fluency on page 8 of the Iowa Academic Standards for Mathematics so the PLC has a common understanding of the expectations for students.
- Which part of the fluency definition was new to you? What does that look like in the classroom?
- Christina said in the second video, “You can’t teach this flexibility. It is caught, right? Kids build this flexibility because they notice things about numbers. They notice 4 is 1 more than 3. I can’t

directly teach that to students, right? I have to help them experience numbers and build their sense of numbers. And when we build that sense of numbers for kids, that builds their flexibility." Do you agree or disagree? Why or why not?
- Share an experience you have had working with students on building their flexibility with number sense.
- What do you hope to learn from engaging in The Flexibility Formula?

Module 1: Understanding

Overview

Module 1 introduces Cognitively Guided Instruction (CGI) to explore children's addition and subtraction strategies, guiding them from direct modeling to more advanced techniques like counting on. Emphasizing the importance of recognizing connections between number facts, the course addresses common challenges students face, such as relying on counting fingers. By identifying various types of number facts and exploring methods to strengthen number relationships, the course seeks to transition students from basic counting to strategic mathematical thinking.

Immersion Experience

The focus of this module is to build educators' own understanding of number sense. A key concept is understanding how Concrete-Representation-Abstract (CRA) models support students as they are developing their number sense. This immersion experience helps educators see the power in combining CRA models to help students build a connected understanding of numbers. This experience includes three activities: 1) Race to 1, 2) Fraction Counting, and 3) Textbook Look Through. The first two activities are recommended, with the third one as optional if you have time.

Approximate Time: 1 – 1.5 hours

Professional Learning Objective: Participants will understand how concrete, representational, and abstract models support student learning of number sense.

Race to 1: CRA in Action with Fractions

This activity focuses on the four number relationships and how you might think you are addressing them, but might not be.

Resources Needed:

- Construction Paper
- Scissors
- Paper & Pencils/Pens
- Fraction Die (wooden cube) – Label 6 faces: 1/2, 1/4, 1/8, 1/16, 1/8, 1/16

Facilitation Instructions

Segment	Instructions
Preparation	1. Watch the Math Game: Race to 1 2. Create the following fraction strips (five colors of construction paper strips, same length: • 1 kept whole • 1 folded and then cut in halves • 1 folded and then cut in fourths • 1 folded and then cut in eighths • 1 folded and then cut in sixteenths *Participants can also make their own fraction strips as the folding is an opportunity to develop key fraction understandings.  *Figure 10: Fraction Strips*
Round 1 Play	**Concrete Only** Everyone plays once through using only the physical fraction strips. ***No recording sheet. No drawing. No writing***. Roll the die, identify the matching strip and place it on your whole strip. The first person to fill to 1 wins. 1. **Roll** the fraction die. 2. **Identify** the strip that matches what you rolled. 3. **Place** it on your whole strip. Pass the die. 4. **Continue adding strips** on each of your turns. 5. **First to reach exactly 1** wins the round.
Round 1 Debrief	**Debrief Prompt:** *"What mathematical understandings did you actually have to use during that game?"* • Guide participants to the honest answer: almost nothing. • They identified which strip matched the fraction on the die – that's it. • No addition. No equivalence. No number sense. Just matching. • **Key follow-up**: *"What would a student actually be learning if this was their fraction activity?"*

Segment	Instructions
Round 2 Play	**Concrete + Representational + Abstract** Play again with the same rules, but now participants also record each turn using both the tape diagram and the number line. **All three representations are active at the same time.** 1. **Roll** the fraction die. 2. **Place** the matching strip on your whole strip (concrete). 3. **Draw** that fraction on your tape diagram – estimate where it falls (representation). 4. **Mark** the running total on your number line and write the fraction (abstract). 5. **The first person to fill 1** wins – but the number line must be kept current. If you don't have your abstract and representation done, you can't win.  *Figure 11: Race to 1 Game*
What to Watch & Listen for in Round 2	**Prompts to use during Round 2:** • "Before you mark your number line, draw it on your tape diagram first." • "How far along are you now? Can you show me that on the number line?" • "Your strip shows ¼. How could you write that using 16ths?" **The moment to highlight:** *When a participant can't update their number line without going back to the concrete strips – that's exactly where learning is happening. Name it for the group.*
Round 2 Debrief	**Debrief Prompt:** *"Now what mathematical understandings were we building?"* • Fraction magnitude – drawing 1/16 forces you to visualize how small it is and compare other quantities you draw to it. • Equivalent fractions – to mark your number line total, you have to reconcile 1/16 + ¼. You can use the strips to see ¼ = 4/16. • Common denominators – same-sized pieces, discovered through the concrete, expressed through the abstract. • Fraction addition – the number line makes the running total visible and necessary. • The concrete doesn't disappear; it becomes the bridge back when the abstract representation gets difficult.

Fraction Counting

Most educators believe counting is easy – and it is, with whole numbers, because we've memorized it. This activity disrupts that comfort. **The goal of Round 1 is not to teach anything.** It's to create a felt contrast. When educators breeze through counting by 2s and then immediately stumble on counting by 2/3s, they experience exactly what students experience with unfamiliar number structures. That discomfort is the entry point for the real learning.

Resources Needed:

- Fraction Counting Slides
  - Animated slide: whole number counting sequence (Round 1)
  - Animated slide: fraction counting sequence (Round 2)
  - Animated slide: abstract with visual representation of each 'improper fraction' (Round 2)

    - Animated slide: mixed number visuals (Round 3 – optional)
- Whiteboard/ chart paper to record any patterns or insights you want to highlight
- Markers

**No manipulatives needed – the slides carry the visual load.*

Key Vocabulary:

Numerator = the count/number of equal-sized parts or fractional units

- Works exactly like whole numbers.
- It tells us how many fractional units we have.
- When we count by 2/3s, the numerator counts: 2, 4, 6, 8 while the denominator stays constant.

Denominator = the denomination or identifies how many equal parts make one whole

- Tells us the size of each piece.
- Like bill denominations in money.
- "Thirds" tells us what the size of items we're counting is. It doesn't change while we count. The size of the items stays the same, and only the number of them changes.

Facilitation Instructions

Segment	Instructions
Preparation	The counting slides have animations built into them. Spend time becoming familiar with the slides and the animations.
Round 1 Play	Let this round feel easy. That's the setup. Don't over-explain or telegraph what's coming. *"Let's warm up with something we probably do a lot. We are going to popcorn count by 2s up to 20. When I point at you, you pop up and say the next number in the sequence."* • Take no more than 20-30 seconds. Keep it brisk. • No debrief yet – move immediately to Round 2. **Round 1 – Count by 2s** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 *Easy. Fast. Feels automatic.*

Segment	Instructions
Round 2 Play	**The goal of this round is to put educators into a state of productive struggle**. You will ***not*** reach 20/3 – stop whenever the group is seeing the pattern and connection to round 1 (usually they start to see it around 8/3 or 10/3, but we keep counting until 20/3). The struggle itself is the lesson. Resist the urge to rescue too quickly. *"Okay, same idea – now we're going to count by two-thirds. Start at zero and go up. Who's got the first number?"* **If the group hesitates or goes quiet:** *"Take your time. What would the first number be if we're counting by two-thirds?"* **If someone says "1 and 1/3" (a mixed number) for 4/3:** *"Great thinking – that's actually correct! But for this round, we're going to leave everything as fractions greater than 1. No converting to mixed numbers yet. So instead of 1 and 1/3, we'd say …4/3. Can you keep going from there?"* This instruction removes the unfamiliarity of switching to mixed numbers. Instead, we want the focus to be on how easy counting with fractions can be ***if*** we help them see the connection to counting whole numbers. **Continue prompting one number at a time when needed:** *" We've got 4/3. What's 2/3 more than that?"* **Slide Cue:** Bring up the animated counting sequence slide. Reveal numbers one at a time as the group produces them – or use it to confirm/correct after the group attempts each step. **Round 2 – Count by 2/3s:** 2/3, 4/3, 6/3, 8/3, 10/3, 12/3, 14/3 … *Stop well before reaching 20/3. The struggle is the point, but also how that struggle goes away once they see the connection to counting whole numbers.*
Round 1 and 2 Debrief	**Debrief Prompt**: *"How were round 1 and round 2 the same? How were they different?"* • The structure was identical – we were still counting forward by a 2. • The difference – it felt unfamiliar; we've never really counted by fractional amounts. • **Key insight to name:** *"Counting with whole numbers is something we do **a lot** with young learners. We don't spend near the amount of time (or maybe even any time) counting with fractions, yet counting is the basis for every operation we do with numbers. If we can't count, then we really aren't adding / subtracting / multiplying / dividing—we are just following steps or memorizing facts. If we want students to operate with understanding, that first starts with actually understanding counting…for all types of numbers."* **Slide Cue:** Bring up the animated visual representation slide. This shows the repetition of the 2/3. You will note that it is not filled in to make wholes because that visual alludes to the mixed number representation. The first visuals connect to the repetition of 2/3.

Segment	Instructions
Round 3 Play (Optional)	**When to use Round 3:** Only if time allows and the group is ready. You can use the final visual slides to show how visuals make the conversion from fractions greater than 1 to mixed numbers feel obvious rather than procedural. Skip it if the Round 2 debrief is still generating strong discussion or you don't have time. Return to the same counting sequence, but now as you go around and count by 2/3s, ask them to change it to a mixed number. You can reveal the **mixed-number visual** alongside each fraction greater than 1. The visual helps them see what the mixed number is. **Slide Cue:** Bring up the animated mixed number visual slide. Reveal each image alongside its fraction greater than 1. Let participants call out the mixed number before it appears.
Round 3 Debrief	**Debrief Prompt:** *"How did having the visual change your ability to see the mixed number?"* **Key Takeaway:** Visuals don't scaffold away understanding - they build it.

Textbook Look Through

Teachers are handed a curriculum and trusted to deliver it. The unspoken assumption is that if a concept matters, it must be in the book. This activity tests that assumption. When teachers examine their own lesson through the lens of the eight number sense concepts and CRA, they don't just hear that something is missing - they see it themselves. That ownership is what makes them open to bringing the missing pieces in. Your job as the coach isn't to point out the gaps. It's to give teachers the lens and let them discover what their students are (and aren't) getting.

Resources Needed:

- A current lesson or chapter from each participant's textbook or curriculum (digital or print)
- Number Sense Concepts Reference Sheet
- CRA Models Quick Reference
- Look-Through Recording Sheet
- Three colors of highlighters or sticky flags (one per category: early numeracy, number relationships, CRA)

Facilitation Instructions

Segment	Instructions
Preparation	Have teachers bring the actual lesson they'll teach next week. Real materials, not samples. You may want to try the activity yourself with the lesson before meeting with your teachers. **Watch for defensiveness**: Some teachers will defend their curriculum because they've invested years in it. Redirect gently: *"We're not evaluating the curriculum. We're evaluating what kids are getting."* The curriculum isn't the enemy; the gap between what's in the book and what kids need is what we are digging into.

Segment	Instructions
Round 1	**The Look-Through (Solo)** Teachers need quiet time to actually examine their materials. Resist the urge to discuss, guide, or walk around making comments during this round. The discoveries they make alone are more powerful than anything you would point out. *"You're going to spend the next 12 to 15 minutes looking through one lesson or chapter from your current curriculum. Your job is to be a detective. For each concept on your reference sheet, ask three questions: Is it here? Where exactly? Is it taught or just used? Use one highlighter color for each category. And pay close attention to what you **don't** find. The blanks are just as important as what's there."* **Pass out the Recording Sheet.** Set a visible timer. No talking during this round. (You can have them fill out the **r**ecording sheet or use Post-it notes or have them use Post-it notes first and then fill out the recording sheet.) **If a teacher finishes early, prompt with:** *"Go back through and check the ones you marked 'found'. Was it really taught or did the lesson just assume kids already had it?"*
Round 2	**Share & Compare** Patterns emerge when teachers compare notes. The goal isn't to evaluate the curriculum or assign blame to publishers. It's to see clearly what students are and aren't getting. Keep the conversation focused on the kids and the experiences kids are getting. Pair teachers up or work in small groups of 3 to 4. Use these prompts to drive the share: • "Which concepts did you find the most of? Which were hardest to find?" • "When you found a concept, was it actually taught or was the lesson assuming kids already had it?" • "Where in the CRA progression does your lesson start? Where does it end? Is that where you think it should start & end?" • "What would happen if a student walked into this lesson without [concept]? Where would they get stuck?" • "Which of the eight concepts didn't show up at all?" **Moments to highlight:** • The teacher who says "I never noticed this isn't in here." That's the awareness shift you're coaching toward - name it for the group. Show the examples where things are missing. • The teacher who finds nothing. That's not a failure of the activity; that's the data. Show the examples of great lessons where nothing needs **to be** modified

Segment	Instructions
Round 1 and 2 Debrief	**Debrief Prompt:** *"What patterns are you noticing across the group?"* • Most lessons lean heavily on abstract and skip past concrete almost entirely or vice versa. They may be concrete-heavy and never give students a chance to see the abstract. • Early numeracy concepts (subitizing, counting, cardinality) are often missing above kindergarten - but our students still need them, especially the ones who came in behind. • Number relationships are frequently implied but rarely taught. The book uses them; the lesson doesn't build them. • Representational models are often a single picture next to the problem, not a sustained bridge between concrete and abstract. **Key follow-up:** *"If a number sense concept isn't in the lesson, what happens to the student who hasn't built it yet?"*
Round 3	**Coaching Next Moves** Do ***not*** let teachers leave with a list of complaints about their curriculum - that's a dead end. They need to leave with one action they can make next week. Small, specific, doable. One concept, one entry point, one move. *"Pick one concept that's missing or light in your lesson. Not all of them - just one. What's one number routine, one game, one question, or one model you could add that would bring that concept in without throwing the lesson out?"* Use the planning table below. Keep it specific. "More subitizing" isn't a plan. "Open tomorrow's lesson with a 5-frame dot flash before the textbook portion" is. **Prompt for teachers who are struggling to start:** *"What's one thing you **could** add Monday morning?"*
Round 3 Debrief	**Debrief prompt:** *"What did this activity change about how you'll look at next week's lesson?"* • Teachers need to understand the intent and design of the HQIM first, examining student evidence, and making purposeful adjustments that preserve coherence, rigor, and the instructional goal. • The teacher's number sense expertise IS super important - the book can't do it for them. • **Key Takeaway:** You're not replacing the curriculum; you're filling in what the curriculum can't. • Coaching shifts from "Here's what you should do" to "What did you notice was missing, and what will you add?"

Deconstruction

The number sense concepts are difficult for educators to place themselves back into the learner mode similar to their students. Counting with whole numbers, for example, is something we all do automatically as we have already built our fluency. So, to place the educator into the learner mode, the activity has them counting with fractions instead. It is important to take a moment and unpack the two or three activities (the third was optional) and explore how those experiences help educators support students. Here are some sample questions that you can use to prompt this discussion:

- *How did the second round of Race to 1 change the mathematical understanding that students have to demonstrate?*
- *How does this connect with other lessons in your classroom and the use of CRA?*
- *How did you feel during the fraction-counting experience?*
- *What would change if we had not started at 0? Would this have made a difference?*
- *How did the visuals support counting with fractions?*
- *How does your learning in The Flexibility Formula align to your current textbook or curricula materials?*

Connection

The connection phase of this module is where educators view the videos and individually reflect by recording their thoughts in The Flexibility Formula Individual Reflection Workbook. Educators should be making connections from the experience at the beginning of the module to content in the videos and reading.

Videos in the Module

Video	Description	PK-2 Length	3-6 Length
Snapshot of Understanding	This video provides an overview of the module.	20:41	23:04
Understanding Versus Doing Math	This video highlights the importance of redefining mathematics to align with the three aspects of rigor: conceptual understanding, procedural skills, and real-life approach.	11:09	10:40
Subitizing	In this video, learn about subitizing, the ability to instantly recognize quantities, and its importance in understanding number relationships. 3rd - 6th grade video extends subitizing into fractions and decimals.	19:22	30:29
Verbal Counting	The video explores the foundational aspects of verbal counting for children. The 3rd-6th grade video extends verbal counting to unit-based counting and place value, helping to deepen the understanding of fractions and multiplication.	17:25	23:47
Object Counting & Cardinality	Explore the foundational concepts of object counting and cardinality, essential for early numeracy. Learn how counting items and understanding their total fosters a strong numerical base. The 3rd-6th grade video extends object counting into fraction counting with a focus on iteration of unit fractions.	21:07	25:51
Spatial Relationships	Explore advanced number relationships that go beyond basic counting, focusing on spatial relationships to enhance reflective thinking about visual patterns and relationships between quantities. The 3rd-6th grade video extends spatial relationships to support multiplication and fractions.	12:37	12:59
One/Two More & Less	This video explores how the number relationship of one/two more and less helps students with their addition facts. The 3rd-6th grade video extends this concept into multiplication and fractions.	13:00	19:29

Video	Description	PK-2 Length	3-6 Length
Benchmark Numbers	This video builds the understanding of how relating numbers to the benchmarks of 5 and 10 supports addition and subtraction strategies and the development of derived facts. The 3rd-6th grade video extends benchmarks to include 1, 2, 5, and 10, or multiplication and fraction benchmarks, to support the understanding of equivalence and operations.	15:38	21:13
Part-Part-Whole	The video explores the "part-part-whole" concept from John Van de Walle's "Teaching Student-Centered Mathematics," emphasizing the importance of understanding numbers as wholes and their relationships.	16:28	29:44
Final Thoughts on Understanding	This video reminds participants that while they looked at each of the eight number sense concepts individually, they are all interconnected.	1:24	1:24
Total		2 hrs. 28 mins	3 hrs. 18 mins

Resources in the Module

- Addition Fact Chart
- Savvy Subitizing Cards
- Visuals to Build Flexibility in PK-2
- Visuals to Build Flexibility in 3-6 (Bonus Module)
- Fraction Subitizing Cards (Bonus Module)
- PK-2 Math Manipulatives
- Disappearing Act Number Routine
- Multiplication Fact Chart

Application

The application portion of the Coaching Routine is located in The Flexibility Formula Individual Reflection Workbook for educators. The application for this module asks educators to try something from this module in their classroom and take notes about what students said or did, what surprised them, and what they might try next. This is an important step where coaches and instructional leaders can support educators as they are trying something new. These experiences will lead to richer PLC conversations.

Reflection in PLCs

PLC time provides an opportunity for educators to share their classroom experiences and support one another in the learning. The following routine leads teachers through sharing and then setting a team goal.

A. Share-Compare-Extend Routine
    a. Share: One classroom observation from your Try-It (see The Flexibility Formula Individual Reflection Workbook).
    b. Compare: What patterns do we notice across classrooms?

c. Extend: What instructional decisions does this lead us to?

B. Group Goal Setting: Based on our discussion, our team will try…
C. End of Module Evidence Wall – Have teachers write one observable student statement, not scores, per Post-it notes or through an electronic collaboration tool. Examples:
    - "Students quickly named 5 when shown dice patterns but needed to count from 6+."
    - "Students explain 9 is 'close to 10' but didn't use it to solve problems."

Lead the PLC through looking for patterns in the student evidence and adjust the team's goal if warranted by the observable data.

Module 2: Observing

Overview

Module 2 focuses on the importance of observing students to gain insights into their learning processes rather than relying solely on assessments. It clarifies the educational purpose of assessment, emphasizing the difference between formative assessments and summative assessments. The module encourages teachers to engage with students through probing questions and discussions, fostering a deeper understanding of their thought processes. Strategies for effective classroom discussions and tasks, particularly in early numeracy, will be provided, along with a reflection on adapting assessment practices to enhance student learning.

Immersion Experience

Module 2 focuses on assessment ***for*** learning. The immersion experiences give educators opportunities to practice observing student learning, analyzing student work, and engaging in PLC data conversations. It is recommended to do the three experiences in order presented below. PLCs may even delay the third experience until after the Connection part of the Coaching Routine which includes watching the videos in the module.

Observing Student Mathematical Thinking

Time: approximately 20 minutes

Professional Learning Objective: Participants will practice observing student mathematical thinking while looking for the eight number sense concepts.

Resources Needed:

- Videos of students sharing their mathematical thinking

Facilitation Instructions

Segment	Instructions
Preparation	Choose an appropriate video of one or more students sharing their thinking. The video could be from any of the following resources: A. Video of student(s) from your school B. Websites: a. Learning Trajectories b. DREME – Development and Research in Early Mathematics Education, Stanford University
Process	Watch the student video together with each educator recording their observations on their own sheet of paper. Educators should make note of the following types of information: • Which of the eight number sense concepts is the student demonstrating understanding. • Evidence of student understanding of the number sense concepts. This could include quotes and/or actions of the student.
Discussion	Lead educators through a discussion using the following questions: • What do you know about the student's understanding of the Number Sense Concepts (subitizing, verbal counting, object counting, cardinality, spatial relationships, benchmarks, one/two more & less, part-part-whole)? • What would you do next instructionally to support the student's learning? **Facilitator Tip:** Help educators focus on an asset-based approach to this activity by refocusing them on what the student does understand and can do. The instructional decisions should build on the student's strengths. This activity may be used throughout the school year when educators need to calibrate their thinking on observing student mathematical thinking focused on the number sense concepts.

Analysis of Student Work Samples

Time: approximately 20 minutes

Professional Learning Objective: Participants will analyze student work looking for evidence of the eight number sense concepts.

Resources Needed:

- Student Samples of Mathematical Work

Facilitation Instructions

Segment	Instructions
Preparation	Ask each educator to bring at least one sample of student mathematics work from a recent lesson and/or assessment.
Process	1. Ask for a volunteer to present the student work they brought and the context. a. Which early numeracy concept(s) was the focus of the sample work? b. Brief description of the learning experience or task. 2. Individual Think Time: a. What do you know about the student's understanding of the number sense concepts (subitizing, verbal counting, object counting, cardinality, spatial relationships, benchmarks, one/two more & less, part-part-whole)? b. What would you do next instructionally to support the student's learning?

Segment	Instructions
Discussion	Lead a whole group discussion debriefing the questions the educators answered individually. Repeat the process for as many samples as time allows. **Facilitator Tip:** Help educators focus on an asset-based approach to this activity by refocusing them on what the student does understand and can do. The instructional decisions should build on the student's strengths. This activity may be used throughout the school year when educators need to calibrate their thinking on analyzing student work focused on the number sense concepts.

PLC Data Conversations

Time: approximately 45 minutes – 1 hour

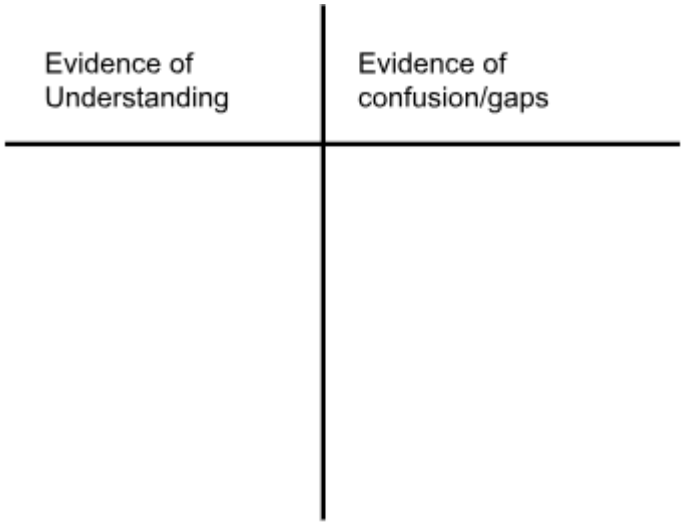
Professional Learning Objective: Participants will analyze student observational data to identify patterns in student learning related to number sense concepts to inform instructional decisions.

Resources Needed:

- PK-2 Early Numeracy Concept Observations Look-Fors (for reference)
- 3-6 Early Numeracy Concept Observations Look-Fors (for reference)
- Teacher observations
- Post-it notes or other tools to capture and sort observational data

Facilitation Instructions

Segment	Instructions
Preparation	Ask teachers to record observations of student thinking during a lesson and bring them to the data conversation. The PLC may choose to bring back observations from the same lesson or from different lessons.
Phase 1 – Share	***Focus:*** What did we observe? Each member of the PLC takes turns sharing the following information: • Which number sense concept(s) did you focus on during observations? • Brief description of the learning experience or task used in the lesson. • General low-inference observations (no interpretation yet). Record individual observations on Post-it notes for the next phase of sorting. Post-it notes are one way to sort observational data, but adjust to a method that works best for your PLC.

Segment	Instructions
Phase 2 - Sort	***Focus:*** What patterns do we see across the four early numeracy concepts? As a team, organize observational data from Phase 1 by concept using a T-chart for each concept.  *Figure 12: T-Chart for Student Evidence*
Phase 3 – Analysis	***Focus:*** What does this mean for instruction? As a PLC use the sorted observational data to answer the following questions: • *Based on our data, which early numeracy concept should be our collective focus?* • *Where are students along the developmental progressions for this concept?* • *What might explain the patterns we are seeing?* (Use a root cause strategy such as the Five Whys)
Phase 4 – Plan	***Focus:*** Now what will we do? As a PLC, answer the following questions: • *Which early numeracy concept will we prioritize?* • *What specific teaching actions will we take?* • *What activities, games, or tasks will help build this concept?* • *What representations will we use?* • *How will we differentiate for different developmental levels?* • *How will we know students are progressing?* Each teacher identifies 1-2 specific actions they will take in their classroom.
Phase 5 – Reflection	***Focus:*** How is this process working? Lead the PLC through a reflection conversation on the data conversation process (Phases 1-4). Here are some sample questions. • *What was the most valuable thing about today's data analysis?* • *What made this collaboration productive?* • *What do we need for our next data conversation?*

Deconstruction

The experiences in this module focused on collecting and analyzing formative assessment data through student observations and work. Every day, students are providing formative assessment data that teachers use to make instructional decisions. These experiences provided opportunities for teachers to practice looking for early numeracy concepts, analyzing the data, and making instructional decisions. Facilitate a conversation after these experiences reflecting on how these processes/experiences support student learning. Here are a few sample questions for this conversation.

- *Which number sense concept(s) were easier to see through observation?*
- *Which number sense concept(s) were easier to see through student work?*
- *How did practicing collecting observation data help you to collect observational data while you are teaching?*
- *What are some ways in which you collected observational data in the moment?*
- *How did analyzing observational data across classrooms support PLC conversations around student learning of early numeracy concepts?*

Connection

The connection phase of this module is where educators view the videos and individually reflect by recording their thoughts in The Flexibility Formula Individual Reflection Workbook. Educators should be making connections from the experience at the beginning of the module to content in the videos and reading.

Videos in the Module

Video	Description	PK-2 Length	3-6 Length
Snapshot of Observing	This video provides an overview of the module.	15:53	15:53
Observing Early Numeracy	This video provides a comprehensive guide to observing and assessing the four early numeracy concepts (subitizing, verbal counting, object counting, and cardinality) identified by Clements and Sarama.	18:36	13:48
Observing the 4 Number Relationships	This video introduces observational assessment tools for John Van de Walle's four number relationships (spatial relationships, one/two more or less, benchmarks, and part-part-whole) designed to inform instructional decisions rather than assign grades.	21:56	16:55
Observing Kids	In this video, participants will observe and assess early mathematical thinking in young children through real examples.	32:45	37:47
Final Thoughts Observing	This video reminds educators to focus on observing students' thought processes alongside their math skills to enhance learning and to utilize assessments to inform instruction and identify individual strengths and challenges.	3:31	3:31
Total		1 hr. 32 mins	1 hr. 27 mins

Resources in the Module

- *NCTM Assessment Research Brief: Five "Key Strategies" for Effective Formative Assessment* by Dylan Wiliam
- 4 Early Numeracy Concept Observation Look-Fors Spreadsheet for PK-2
- 4 Early Numeracy Concept Observation Look-Fors Spreadsheet for 3-6
- Assessment for the 4 Number Relationships PK-2
- Assessment for the 4 Number Relationships 3-6
- Missing Part Cards PK-2
- Missing Part Cards 3-6
- Number Sense Visuals to Build PK-2
- Visuals to Build Flexibility in 3-6 (Bonus Module)

Application

The application portion of the Coaching Routine is located in The Flexibility Formula Individual Reflection Workbook for educators. The application for this module asks educators to try something from this module in their classroom and take notes about what students said or did, what surprised them, and what they might try next. This is an important step where coaches and instructional leaders can support educators as they are trying something new. These experiences will lead to richer PLC conversations.

Reflection in PLCs

PLC time provides an opportunity for educators to share their classroom experiences and support one another in the learning. The following routine leads teachers through sharing and then setting a team goal.

A. Share-Compare-Extend Routine
    a. Share: One classroom observation from your Try-It (see The Flexibility Formula Individual Reflection Workbook).
    b. Compare: What patterns do we notice across classrooms?
    c. Extend: What instructional decisions does this lead us to?
B. Group Goal Setting: Based on our discussion, our team will try…
C. End of Module Evidence Wall – Have teachers write one observable student statement, not scores, per Post-it notes or through an electronic collaboration tool. Examples:
    - "Students quickly named 5 when shown dice patterns but needed to count from 6+."
    - "Students explain 9 is 'close to 10' but didn't use it to solve problems."

Lead the PLC through looking for patterns in the student evidence and adjust the team's goal if warranted by the observable data. This shifts teacher thinking toward **assessment for learning**, exactly as described in the Observing module videos.

Module 3: Connecting

Overview

Module 3 explores the interconnectedness of the eight number sense concepts with broader mathematics principles, emphasizing a holistic approach to teaching mathematics. This module advocates for integrating various models and strategies, aligning with educational standards while focusing on foundational skills. It highlights the importance of curriculum examination for deeper understanding and encourages real-life applications of mathematics.

Immersion Experience

Sometimes educators get tunnel vision and are only familiar with the standards and/or lessons they have taught each year. The beauty of mathematics is how the concepts are connected and not isolated skills. It is important for educators to take time and explore how the standards in their grade level connect to the early numeracy concepts, other standards in their grade level and standards in other grade levels.

Time: approximately 30-45 minutes (for the first time completely through); approximately 20 minutes only doing the web specific portion of the activity

Professional Learning Objective: Participants will make connections between mathematics standards and early numeracy concepts.

Resources Needed:

- Iowa Academic Standards for Mathematics, pp. 3-8
- Iowa Academic Standards for Mathematics Session 2: Focus Screen Cast
- Iowa Academic Standards for Mathematics Session 2: Slide Deck
- Iowa Academic Standards for Mathematics Session 3: Coherence Screen Cast
- Iowa Academic Standards for Mathematics Session 3: Slide Deck
- Blank Web
- Tondevold's Scope & Sequence PK-2
- Tondevold's Scope & Sequence 3-6
- Achieve the Core's Coherence Map
- Learning Trajectories

Facilitation Instructions

Segment	Instructions
Preparation	Review the following resources on the shift of Focus: • Iowa Academic Standards for Mathematics, pp. 3-8 • Iowa Academic Standards for Mathematics Session 2: Focus Screen Cast • Iowa Academic Standards for Mathematics Session 2: Slide Deck • Iowa Academic Standards for Mathematics Session 3: Coherence Screen Cast • Iowa Academic Standards for Mathematics Session 3: Slide Deck
Understanding Focus	One of the instructional shifts in the Iowa Academic Standards for Mathematics is Focus, which requires prioritizing the Major Work of the grade level. The major work of the grade ensures students establish robust foundations, solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply mathematical concepts to real-world problem-solving scenarios. Read pp. 3-8 in the Iowa Academic Standards for Mathematics. Utilize either the Session 2: Focus Screen Cast or Slide Deck to help educators develop an understanding around the shift of Focus.
Round 1 – Starting the Web	The goal is to help educators focus on the major work of the grade level and make connections to everything they have been learning about the early numeracy concepts. As a coach, you may need to ask questions during this activity if they are stuck making the connections. ***Hand out the blank web handout*** (See Appendix). 1. Select a major cluster in your grade level and place that in the center of your web. 2. Use the following resource to help you complete the web diagram by connecting early number concepts and relationships to your major cluster. a. Tondevold's Scope & Sequence 3. When everyone is done, have each person or group share their web. ***Facilitator Tip:*** A completed web example is provided in the Appendix. This example may be useful as educators work through this process for the first time. *After you have taken educators through this complete activity once, you can repeat this portion throughout the year with a focus on other major clusters.*
Understanding Coherence	Another shift in the Iowa Academic Standards for Mathematics is Coherence, which requires coherence within and across grade levels to ensure instruction follows a logical mathematics progression. Read p. 8 in the Iowa Academic Standards for Mathematics. Use either the Session 3: Coherence Screen Cast or Slide Deck to help teachers understand the shift of coherence. ***Facilitator Tip***: If your teachers have never worked with the Achieve the Core's Coherence Map, it would be best to spend some time orienting teachers to the tool. The same applies to the Learning Trajectories for grades PK-3.

Segment	Instructions
Round 2 - Finishing the Web	The goal of this round is for educators to make connections vertically outside their grade level. Ask educators to ***add to their web*** they started in Round 1 using a different color and the following tools: • Achieve the Core's Coherence Map • The Learning Trajectories Educators should add the following to their web: • Other standard clusters within the grade level that connect to the selected cluster. • Standard clusters from the previous grade that are connected to the selected cluster • Standard clusters from the next grade that are connected to the selected cluster. This helps educators to understand horizontal and vertical coherence. *After you have taken educators through this complete activity once, you can repeat this portion throughout the year with a focus on other major clusters.*

Deconstruction

The focus of this module is to make connections between the early numeracy concepts and other mathematical concepts. Educators usually focus on understanding on the standard(s) they are teaching. It is just as important that educators understand how standards connect to each other within the grade level and to other grade levels. Otherwise, we risk the mathematical concepts being taught as isolated skills instead of cohesive and connected concepts. Lead educators through a discussion unpacking how the web activity supported their understanding in making connections between standards and early numeracy concepts. Here are some sample questions to use.

- *What was new learning for you as you created the web?*
- *What did the web activity affirm in your understanding of mathematical concepts?*
- *How does the web activity help you make instructional decisions to support your students?*
- *What questions do you have from doing the web activity?*
- *What mathematical concepts would you like to investigate further as a result of this activity?*

Connection

The connection phase of this module is where educators view the videos and individually reflect by recording their thoughts in The Flexibility Formula Individual Reflection Workbook. Educators should be making connections from the experience at the beginning of the module to content in the videos and reading.

Videos in the Module

Video	Description	PK-2 Length	3-6 Length
Snapshot of Connecting	This video provides an overview of the module.	19:57	19:57

Video	Description	PK-2 Length	3-6 Length
Connecting the Number Concepts	This video demonstrates how the eight number sense concepts work together as an interconnected system rather than isolated skills.	22:29	31:59
Connecting to Other Math Concepts	This video introduces a comprehensive mathematics scope and sequence document that maps how the eight number sense concepts underpin all mathematical learning in PK-2nd grade. The 3rd-6th grade video explores a comprehensive approach to number sense that connects foundational concepts to broader mathematical ideas.	28:33	19:13
Connecting to Home	"Math at home" letters are provided to help caregivers understand essential numeracy skills and activities that can be integrated into daily life.	2:51	2:51
Final Thoughts on Connecting	Discover how recognizing the interrelation of number sense concepts can enhance learning, and learn strategies for building a solid foundation while managing content coverage effectively.	2:44	2:44
Total		1 hr. 16 mins	1 hr. 16 mins

Resources in the Module

- Constance Kamii's Web of Understanding
- Graham Fletcher's Trajectories for the Effective Learning and Teaching of Mathematics in the Primary Grades (K-2)
- Content Focus Areas by Student Achievement Partners - Use the Iowa Academic Standards for Mathematics instead
- PK-2 Math Scope & Sequence with Number Sense Concepts Embedded
- Achieve the Core's Coherence Map
- Clements and Sarama's Learning Trajectories
- Math at Home Letters
- Directions for Farkle
- 3rd - 5th Grade Math Scope & Sequence with Number Sense Concepts Embedded

Application

The application portion of the Coaching Routine is located in The Flexibility Formula Individual Reflection Workbook for educators. The application for this module asks educators to try something from this module in their classroom and take notes about what students said or did, what surprised them, and what they might try next. This is an important step where coaches and instructional leaders can support educators as they are trying new. These experiences will lead to richer PLC conversations.

Reflection in PLCs

PLC time provides an opportunity for educators to share their classroom experiences and support one another in the learning. The following routine leads teachers through sharing and then setting a team goal.

A. Share-Compare-Extend Routine
    a. Share: One classroom observation from your Try-It (see The Flexibility Formula Individual Reflection Workbook).
    b. Compare: What patterns do we notice across classrooms?
    c. Extend: What instructional decisions does this lead us to?
B. Group Goal Setting: Based on our discussion, our team will try…
C. End of Module Evidence Wall – Have teachers write one observable student statement, not scores, per Post-it notes or through an electronic collaboration tool. Examples:
    - "Students quickly named 5 when shown dice patterns but needed to count from 6+."
    - "Students explain 9 is 'close to 10' but didn't use it to solve problems."

Lead the PLC through looking for patterns in the student evidence and adjust the team's goal if warranted by the observable data.

Module 4: Experiencing

Overview

Module 4 introduces a fundamental shift from traditional mathematics instruction to providing rich mathematical experiences for students. Everything covered in the course so far -- building number sense, student flexibility, and teaching foundations—forms the "iceberg beneath the surface" that supports meaningful mathematical experiences. Rather than simply teaching lessons to impart knowledge, the goal is to let students experience mathematics firsthand, similar to experiencing a rollercoaster rather than just hearing about it.

Immersion Experience

There are three types of experiences students should engage in regularly to support the building of their number sense: number sense routines, contextual problems, and practice. This immersion experience includes three activities: 1) number sense routines, 2) contextual problems, and 3) experience review. The first two activities provide opportunities for educators to engage in the experiences first as a student and then examine the same experiences through an educator's lens. The third activity provides an opportunity for educators to find these experiences in their own curriculum materials.

Number Sense Routines

Time: approximately 20 minutes

Professional Learning Objective: Participants will engage in number sense routines that promote reasoning and make student thinking visible.

Required Resources:

- Splat!
- Esti-Mystery
- Subitizing Slides
- Math Flips
- Notice and Wonder and "How many?"

Facilitation Instructions

Segment	Instructions
Preparation	Choose which of the number sense routines you want educators to experience as students from the list below: • Splat! • Esti-Mystery • Subitizing Slides • Math Flips • Notice and Wonder and "How many?" Each link above includes slide decks, explanations and directions for the number sense routine.
Experience	None of the number sense routines above require pencil and paper to participate in. Actually, the goal is for students to engage mentally individually and then engage with other students. Use the slides and the resources provided for the selected number sense routine to engage educators in the routine as students. ***Facilitator Tip:*** Before you start the number sense routine explain you are asking educators to engage in the activity like students would. There will be a time after engaging in the routine to step back and discuss this from the educator's lens.
Debrief	Facilitate a debriefing conversation focused on understanding how the routine supports building a student's number sense. Here are some sample questions. • *What strategies surfaced?* • *How does this kind of routine support flexible thinking in students?* • *What experiences have you had using subitizing cards (insert other number sense routine) in class with students?*

Contextual Problems

Time: approximately 30 minutes

Professional Learning Objective: Participants will explore contextual problems that support conceptual understanding and productive struggle.

Required Resources:

- Numberless Word Problems
- Robert Kaplinsky "How Old is the Shepherd?" video
- Iowa Academic Standards for Mathematics

Facilitation Instructions

Segment	Instructions
Preparation	Choose a numberless word problem for the experience. The website includes slide decks, explanations, and directions on how to use of the numberless word problems.
Contextual Problems	Show the Robert Kaplinsky "How Old is the Shepherd?" video. Facilitate a discussion based on what happened in the video. Here are some sample questions. • *Why do you think most students automatically try to add, subtract, or divide the numbers instead of saying the problem makes no sense?* • *What insight does this video give us about how we teach solving word problems?* • *How might we help students pause and consider if the answer they got makes sense?* ***Moment to highlight***: This video highlights how students see a word problem and become answer getters instead of trying to understand the problem. Students will sometimes choose whatever operation you have been working on in class whether it makes sense or not. Another issue to note is that students may rely on keywords rather than key concepts when solving problems. This issue with keywords is that some keywords can indicate more than one operation and therefore students choose the wrong operation. ***Example:*** Natalie has saved $38. This $25 less than her sister Holly has saved. How much money does Holly have? The key words "less than" may lead students to use subtraction even though this problem requires addition. ***Review the Key Concepts Table*** located in Appendix A of the Iowa Academic Standards for Mathematics.
Experience	***Facilitator Tip:*** Before you start the number sense routine explain you are asking educators to engage in the activity as students would. There will be a time afterwards to step back and discuss the experience through the educator's lens. ***Model a numberless word problem*** Ask educators questions at each stage of the problem similar to those you would ask students. The website includes the questions to ask.
Debrief	Facilitate a debriefing conversation focused on understanding how the numberless word problems support building a student's problem-solving skills. Here are some sample questions. • *How would numberless problems help students solve word problems?* • *How have you helped students to solve word problems?*

Experiences Review

Time: approximately 20-30 minutes

Professional Learning Objective: Participants will review current curriculum materials to identify the three types of experiences (number sense routines, contextual problems, and practice) that support the development of students' number sense.

Required Resources:

- Lesson from current curriculum materials
- Sample Lesson from IM as an example with notes - IM Grade 1 Unit 3 Lesson 3 and annotations of the lesson

Facilitation Instructions

Segment	Instructions
Preparation	Before the session, look through the curriculum the teachers use to find a lesson that has a routine, story problems, and practice right in the lesson. If you cannot find a lesson to discuss that has all three, use the lesson to discuss how the missing experience could be added.
Process	Start the experience by giving educators the lesson and asking them to work together to determine whether the lesson contains a Number Sense Routine, Contextual Problem, and Practice. ***Facilitator Tip:*** The Iowa Academic Standards for Mathematics clarify that practice must be meaningful (builds on and extends understanding), purposeful (links to curriculum goals and targets an identified need based on multiple sources) and distributed (consists of short periods of systematic practice spread over a long period). Walk around and listen to the conversations. Make note of any that you want to elevate to the whole group discussion. If there are all three experiences in the lesson, spend some time discussing if they think they are good or if there are tweaks they think should be made to make them better. If one or more of the experiences are missing, discuss what that experience could look like in this lesson and ***if*** it should be added in. The experience selected should depend on the lesson's purpose and where it falls within the progression of learning. Students may not need a particular routine or type of practice if they have already demonstrated the intended understanding.
Sample Lesson	IM Grade 1 Unit 3 Lesson 3 and annotations of the lesson This lesson ***has a Quick Images routine*** already as the warm-up. As you go through the lesson, the entire lesson is basically ***practice with their facts***, just in a new situation (analyzing equivalence). *Should we add more practice with equality though? Is it needed in this lesson, or is this more of an introduction?* ***The piece that is missing is a contextual situation.*** Activities you could do with your teachers to adjust the lesson so it has all three: ***Option 1*** – Have the teachers create an example themselves and then spend time looking at the examples and discussing them. ***Option 2*** – Give educators a prepared example that could be added to the lesson. Discuss where it would fit and how they think it will support students' learning. *"Sierra has 5 red toy cars and 2 blue toy cars. Sam has 1 red toy car and 5 blue toy cars. Do they have the same number of cars?"*

Deconstruction

The purpose of the first two experiences was to place educators in the student role first and then to examine the experiences from the educator role. These activities focus on the first two types of experiences students should engage in on a regular basis: number sense routines and contextual

problems. After finishing the experiences, lead educators through a discussion unpacking the experiences and how they support student learning. Here is a sample set of questions.

- *How do the number sense routines support student flexible thinking through the eight number sense concepts (subitizing, verbal counting, object counting, cardinality, spatial relationships, benchmark numbers, one/two more & less, part-part-whole)?*
- *How do contextual problems support student learning?*
- *How does both the number sense routines and contextual problems connect with the Standards for Mathematical Practice?*

Connection

The connection phase of this module is where educators view the videos and individually reflect by recording their thoughts in The Flexibility Formula Individual Reflection Workbook. Educators should be making connections from the experience at the beginning of the module to content in the videos and reading.

Videos in the Module

Video	Description	PK-2 Length	3-6 Length
Snapshot of Experiencing	This video provides an overview of the module.	4:26	4:26
Why These 3 Types of Experiences	This video reveals three research-backed types of mathematical experiences that should be part of every student's learning, based on findings from the IES practice guide "Assisting Students Struggling with Mathematics: Intervention in Elementary Grades".	25:00	25:00
Number Sense Routines	This video introduces number sense routines as the first essential mathematical experience, emphasizing that these are repeatable instructional structures focused on building the eight number sense concepts rather than procedures.	20:11	25:04
Contextual Problems	This video explores using contextual (story/word) problems as the second essential mathematical experience, drawing from Cognitively Guided Instruction principles that children should begin with intuitive problem-solving before learning mathematical symbols.	26:15	31:23
Practice	This video explores purposeful practice as the third essential mathematical experience for developing specific skills and ensuring students grasp concepts before engaging in independent practice.	23.:04	20:40
Using Timed Activities	This video gives practical guidance on using short, non-punitive timed activities and games to build students' arithmetic fluency.	11:58	11:58
Modifying Textbook Lessons	This video provides practical solutions for teachers who feel constrained by textbook requirements but want to create meaningful mathematical experiences for students.	16:16	16:16
Final Thoughts on Experiencing	Enhance your mathematics teaching approach with a focus on flexibility and confidence building for your students. Discover the gradual yet rewarding transition to flexible teaching methods, where teachers act as "spotters" to nurture problem-solving skills.	14:08	14:08

Video	Description	PK-2 Length	3-6 Length
Total		2 hrs. 21 mins	2 hrs. 28 mins

Resources in the Module

- *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools*, Institute of Education Sciences (IES) 2009
- *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades*, IES 2021
- Helping Kids Build Flexibility Through Questioning
- Keywords Versus Key Concepts
- Problem Types Charts (also see Appendix A in the Iowa Academic Standards for Mathematics)
- PK-2 Games
- What is the Concrete Representational Abstract Approach video by Christina Tondevold
- The Flexibility Formula: The Math Experiences Planner
- Content Focus Areas by Student Achievement Partners - Use the Iowa Academic Standards for Mathematics instead
- Productive Struggle video by Robert Kaplinsky
- 3rd - 6th Grade Games

Application

The application portion of the Coaching Routine is located in The Flexibility Formula Individual Reflection Workbook for educators. The application for this module asks educators to try something from this module in their classroom and take notes about what students said or did, what surprised them, and what they might try next. This is an important step where coaches and instructional leaders can support educators as they are trying something new. These experiences will lead to richer PLC conversations.

Reflection in PLCs

PLC time provides an opportunity for educators to share their classroom experiences and support one another in the learning. The following routine leads teachers through sharing and then setting a team goal.

A. Odd one Out
    a. Consider these terms: number sense routine, contextual problem, practice opportunities, and questions
    b. Each person in the PLC briefly explains one of the terms.
    c. Individually, determine which of the terms is the “odd one out”, or the term that has an attribute the other 3 do not, is missing an attribute the other 3 do have, is in a different category than the other 3, etc.
    d. Each person shares and justifies their odd one out choice.
B. Share-Compare-Extend Routine
    a. Share: your thought bubble pages
    b. Compare: What patterns do we notice across classrooms?
    c. Add a thought bubble your group members shared that also applies to your experience.

d. Extend: What instructional decisions does this lead us to?

C. Group Goal Setting: Based on our discussion, our team will try…

D. End of Module Evidence Wall – Have teachers write one observable student statement, not scores, per Post-it note or through an electronic collaboration tool. Examples:
   - "Students quickly named 5 when shown dice patterns but needed to count from 6+."
   - "Students explain 9 is 'close to 10' but didn't use it to solve problems."

Lead the PLC through looking for patterns in the student evidence and adjust the team's goal if warranted by the observable data.

Instructional Coaching Resources

The following set of instructional coaching resources may be used by instructional coaches, administrators and instructional leaders as they support educators' growth in integrating number sense in their daily mathematics instruction.

Coaching Stems

The following coaching questions may be used as you work with educators to help them reflect on number sense instructional practices.

- Which number sense concepts – subitizing, one-to-one, cardinality, or verbal counting – are you hoping to see your students build today?
- How will you know students are moving from 'just counting' to showing flexible number thinking?
- I noticed students using _____. What did you notice?
- When students got the last item, how did you see them make sense of the total?
- What strategies did you see students use besides counting each one?
- Where did you see flexibility?
- What routine could you repeat to strengthen subitizing in your class?
- How might you help students shift from one-to-one matching toward understanding cardinality?
- What opportunities could you add for students to practice verbal counting in meaningful contexts?
- What questions could you ask to push beyond 'How many?' to 'How did you know?'

Instructional Look-Fors

The following are a set of look-fors for each number sense concept. The look-fors include both teacher and student actions. They are not all inclusive. Feel free to add other look-fors created in collaboration with teachers to your own list.

Subitizing

- Teacher uses quick images (dot cards, dice patterns, ten-frames, etc.) with limited exposure time.
- Students explain *what they saw first* and *how they knew the total*.
- Teacher encourages multiple strategies.
- Students begin to chunk or group rather than count each item.

Cardinality

- Teacher emphasizes that the last number said represents "*how many in all*."
- Students demonstrate understanding by not needing to recount to answer "How many?"

- Teacher asks follow-up questions: “So, how many are there altogether?” “If we added one more, how many are there altogether?”
- Students can add one more and state the new total without starting over.

Object Counting and One-to-One Correspondence

- Teacher provides manipulatives and prompts students to match one object to one number word.
- Students consistently point, touch, or move items as they count.
- Teacher redirects when students skip or double-count.
- Activities (like counting collections) provide varied practice.

Verbal Counting

- Teacher provides daily counting practice.
- Students attempt stable order of numbers, even if errors occur.
- Teacher models and scaffolds tricky transitions (e.g., 9,10; 19, 20).
- Counting activities connect to meaningful contexts.

Spatial Relationships

- Teacher uses structured visual representations (dot cards, ten-frames, Rekenreks, etc.) and intentionally varies arrangements.
- Teacher asks questions like, “What did you see? How did you see it?” to prompt visualization and multiple perspectives.
- Teacher encourages students to explain *how* they know, not just *how many*.
- Teacher highlights different spatial configurations that represent the same quantity to build flexibility.
- Teacher models grouping and pattern recognition (e.g., “I see two rows of three – that’s six”).

One/Two More and Less

- Teacher engages students in quick routines comparing consecutive numbers.
- Teacher encourages reasoning without recounting (“How do you know it’s one more?”).
- Students use visuals such as number lines, ten-frames, or counters to show quantities increase or decrease by one or two.
- Students draw connections between these relationships and early addition/subtraction expressions

Benchmarks of 5 and 10

- Teacher regularly uses five- and ten-frame models to organize thinking and encourage pattern recognition.
- Teacher highlights 5 and 10 as key reference points.
- Teacher encourages students to describe other numbers in relation to benchmarks.
- Teacher models strategic thinking that builds from benchmarks.

Part-Part-Whole

- Teacher uses manipulatives and visuals to model decomposition and recomposition.
- Teacher poses open problems that invite multiple part-whole relationships.
- Teacher encourages discussion around strategies that reveal structure.
- Students connect part-whole thinking to early operation symbols and stories.

Instructional Shifts for Building Early Numeracy and Number Relationships PK-2

This reflective tool assists teachers in assessing their current instructional practices related to foundational number sense concepts and planning next steps for growth.

How to Use This Tool

1. Self-assess your current practice for each concept by placing an X on the arrow continuum based on the descriptors below.
2. Reflect on classroom evidence and not examples.
3. Identify and plan next steps from the "Example Actions Steps" below.

Subitizing

Starting	Building	Implementing
Uses quick images but primarily checks for correct answers or counting ability.	Facilitates short "flash" experiences and prompts students to describe what they saw.	Regularly engages students in flexible subitizing, prompting reasoning ("How did you see it?") and connecting visual patterns to composition/ decomposition strategies.

Example Action Steps for Building Early Numeracy:

- Use quick images (dot cards, ten-frames, dice patterns) for short exposure
- Ask students to describe what they saw and how they knew.
- Encourage multiple ways to see the total (e.g., 6 as 3+3, 4+2, 5+1).

Student Evidence	Notes	Reflection

Object Counting and One-to-One Correspondence

Starting	Building	Implementing
Provides counting opportunities with manipulatives but focuses on accuracy of count.	Observes and scaffolds when students skip or double-count, uses authentic contexts (snacks, toys, collections).	Intentionally varies groupings and tasks to deepen understanding of quantity and connects one-to-one correspondence to cardinality and comparison.

Example Action Steps for Building Early Numeracy:

- Provide opportunities to count with manipulatives, matching each object to one number word.
- Model matching actions to words and redirect when students skip or double count.
- Use counting collections to give frequent practice in varied contexts.

Student Evidence	Notes	Reflection

Cardinality

Starting	Building	Implementing
Focuses on rote counting to reach a final number.	Begins emphasizing "the last number tells how many." Uses prompts to check understanding.	Invites students to flexibly add one more, reason about totals, and generalize the concept across contexts.

Example Action Steps for Building Early Numeracy:

- Emphasize that the last number said tells "how many" in all.
- Ask follow-up questions like "How many are there altogether?"
- Invite students to add one more and state the new total without recounting.

Student Evidence	Notes	Reflection

Verbal Counting

Starting	Building	Implementing
Leads whole-group counting songs and chants.	Uses daily routines to reinforce number sequence and tricky transitions.	Connects counting to quantity, patterns, and operations ("What happens when we count back two?"). Encourages student-led counting experiences.

Example Action Steps for Building Early Numeracy:

- Integrate daily counting (choral counting, counting around the circle, number games, etc.).
- Support tricky transitions (9 to10, 19 to 20) with modeling and visuals.
- Connect counting to real-life contexts (snacks, steps, manipulatives).

Student Evidence	Notes	Reflection

Spatial Relationships

Starting	Building	Implementing
Displays dot cards and ten-frames but uses them primarily for counting practice.	Encourages students to notice and describe spatial patterns ("I saw two rows of three").	Designs experiences where students compare structures, mentally visualize arrangements, and discuss efficiency of visual groupings.

Example Action Steps for Building Number Relationships:

- Use five- and ten-frames to support seeing numbers without counting.
- Model structured arrangements of dots or objects.
- Ask: "How did you see it?" to encourage flexible grouping.
- Connect visual routines to abstract number equations.

Student Evidence	Notes	Reflection

One/Two More and Less

Uses counting to determine "one more" or "one less."	Introduces number paths, number lines, Rekenreks, or visuals to represent change in quantity.	Encourages reasoning without recounting, and connects the reasoning to addition and subtraction symbols and relational thinking.

Example Action Steps for Building Number Relationships:

- Play games comparing numbers or building “one more/less” towers.
- Use number paths to visualize moving up or down.
- Encourage students to reason: “If I had one more, what would I have?” “If I had one more ten, what would I have?”

Student Evidence	Notes	Reflection

Benchmarks of 5 and 10

Starting	Building	Implementing
Uses ten-frames and manipulatives but does not highlight relationships to 5 and 10.	Uses five and ten-frames along with other manipulatives to provide students with opportunities to visualize the benchmarks of 5 and 10 in counting, addition and subtraction.	Designs tasks where students flexibly compose/decompose around 5 and 10 and apply this reasoning to mental math.

Example Action Steps for Building Number Relationships:

- Anchor understanding with 5- and 10-frames.
- Model decomposing and composing numbers around 5 and 10.
- Encourage students to use these as reference points in mental math.

Student Evidence	Notes	Reflection

Part-Part-Whole

Starting	Building	Implementing
Introduces part-whole tasks (e.g., “make 5”) using manipulatives.	Encourages students to find multiple combinations and talk about their thinking.	Intentionally connects part-whole reasoning to equations, story problems, and relational understanding of addition and subtraction.

Example Action Steps for Building Number Relationships:

- Engage students in finding different ways to make a number.
- Use manipulatives and equations to connect visual and symbolic representations.
- Highlight inverse relationships (e.g., 3+4=7, and 7-4=3).

Student Evidence	Notes	Reflection

Reflection Prompts

- Which number sense concept or relationship do your students engage with most naturally?
- What evidence shows students are reasoning, not just counting?
- What small instructional shift could strengthen your students' number sense next week?

Instructional Shifts for Building Early Numeracy and Number Relationships 3rd – 6th Grade

This reflective tool assists teachers in assessing their current instructional practices related to foundational number sense concepts and plan next steps for growth.

How to Use This Tool

1. Self-assess your current practice for each concept by placing an X on the arrow continuum based on the descriptors below.
2. Reflect on classroom evidence and not examples.
3. Identify and plan next steps from the "Example Actions Steps" below.

Subitizing

Starting	Building	Implementing
Uses visuals or arrays for counting or recall of facts.	Encourages students to see groups within arrays or decompositions (e.g., 7 x 8 as 7 x 5 + 7 x 3).	Promotes flexible structuring and decomposition of numbers in mental computation, fractions, and area models.

Example Action Steps for Building Early Numeracy:

- Use dot images, arrays, and decompositions to explore multiple ways to see the same number.
- Ask "What did you see? How do you know?" during computation.
- Model flexible structuring during mental math.

Student Evidence	Notes	Reflection

Object Counting and One-to-One Correspondence

Starting	Building	Implementing
Treats quantities as countable objects only.	Connects counting and unitizing to place value and equivalence (e.g., 1 ten = 10 ones).	Encourages reasoning about the same quantity in different forms (e.g., 0.5 = ½ = 50%).

Example Action Steps for Building Early Numeracy:

- Use manipulatives and base-ten blocks to represent the same quantity in multiple forms (e.g., 1 ten = 10 ones).
- Connect equivalent representations across whole numbers, fractions, and decimals.
- Encourage students to explain how they know two amounts are equal.

Student Evidence	Notes	Reflection

Cardinality

Starting
Focuses on finding answers (totals, counts).

Building
Encourages students to interpret numbers as quantities in context.

Implementing
Promotes reasoning about relative size, magnitude, and relationships between quantities (e.g., “How many times larger?”).

Example Action Steps for Building Early Numeracy:

- Ask students to describe what a number represents in context.
- Use comparison and multiplicative-comparison questions, such as “how many times as large or small” questions.
- Model reasoning about magnitude and relationships, not just totals.

Student Evidence	Notes	Reflection

Verbal Counting

Starting
Emphasizes skip-counting and memorization of facts.

Building
Connects skip-counting to multiplicative structure and repeated reasoning (e.g., connecting 1/8 + 1/8 + 1/8 to 3(1/8)).

Implementing
Encourages flexible use of patterns in place value, scaling, and ratio reasoning.

Example Action Steps for Building Early Numeracy:

- Build fluency through skip-counting connected to arrays or ratio tables.
- Encourage seeing structure in multiples and patterns.
- Pose questions such as “If I double one factor, how does the product change?”

Student Evidence	Notes	Reflection

Spatial Relationships

Starting	Building	Implementing
Uses area models procedurally (for algorithms).	Connects spatial models (arrays, area) to partial products and distributive reasoning.	Designs tasks where students use spatial structure to reason about number relationships, equivalence, and efficiency.

Example Action Steps for Building Number Relationships:

- Use area models, number lines, and arrays to visualize number relationships.
- Encourage flexible representations of the same structure.
- Invite students to compare and connect visual models to symbolic reasoning.

Student Evidence	Notes	Reflection

One/Two More and Less

Starting	Building	Implementing
Treats addition/subtraction as isolated operations.	Highlights relationships among numbers and operations (e.g., “if 8 x 7 = 56, what is 8 x 8?).	Encourages use of known relationships and structure to generalize patterns and reason algebraically (e.g., How does knowing 30 x 15 = 450, help us solve 29 x 15?).

Example Action Steps for Building Number Relationships:

- Pose open-number sentences (e.g., “What could go in the box to make this true?”).
- Encourage students to reason using known relationships (“If 8 x 7 = 56, what is 8 x 8?”).
- Highlight patterns and generalizations.

Student Evidence	Notes	Reflection

Benchmarks

Starting	Building	Implementing
Uses rounding and benchmark numbers only for estimation.	Encourages reasoning with friendly numbers and proportions (e.g., half of 10, double 50).	Promotes fluency with flexible benchmarks for fractions, decimals, and percentages (e.g., 25%, 50%, 75%).

Example Action Steps for Building Number Relationships:

- Model rounding and estimating with purpose (“Is it closer to 5 or 10?”).
- Encourage students to reason with friendly numbers in computation and fraction comparison.
- Use 25%, 50% and 75% as benchmarks for reasoning with fractions and decimals.

Student Evidence	Notes	Reflection

Part-Part-Whole

Starting	Building	Implementing
Focuses on finding missing numbers in basic operations.	Connects part-part-whole relationships to fact families and equations.	Encourages reasoning about equivalence, proportionality, and balance.

Example Action Steps for Building Number Relationships:

- Use open-ended composition and decomposition tasks.
- Highlight relationships among equations (fact families, inverse operations).
- Connect part-part-whole reasoning to solving equations and balancing expressions.

Student Evidence	Notes	Reflection

Reflection Prompts

- Which number sense concept or relationship do your students engage with most naturally?
- What evidence shows students are reasoning, not just counting?
- What small instructional shift could strengthen your students' number sense next week?

Appendix

Number Sense Concepts Reference Sheet

Number Sense Concept	Brief Description
Subitizing	Quickly recognizing and naming the quantity in a group without counting
Verbal Counting	Saying numbers out loud in a specific, ordered sequence
Object Counting	Finding and naming the total number items in a group or collection
Cardinality	Knowing that the final number counted stands for the total size of the entire group
Spatial Relationships	Recognizing quantities and number relationships through the spatial arrangement of objects or visual patterns.
One/Two More & Less	Knowing what is one or two more or less than a given quantity. Visual representations can support this understanding.
Benchmarks	Understanding how numbers relate to familiar reference points, such as 5 and 10, and later to benchmark fractions and decimals.
Part-Part-Whole	Understanding that a whole is composed of parts and can be decomposed and recomposed in different ways.

Concrete-Representational-Abstract Reference Sheet

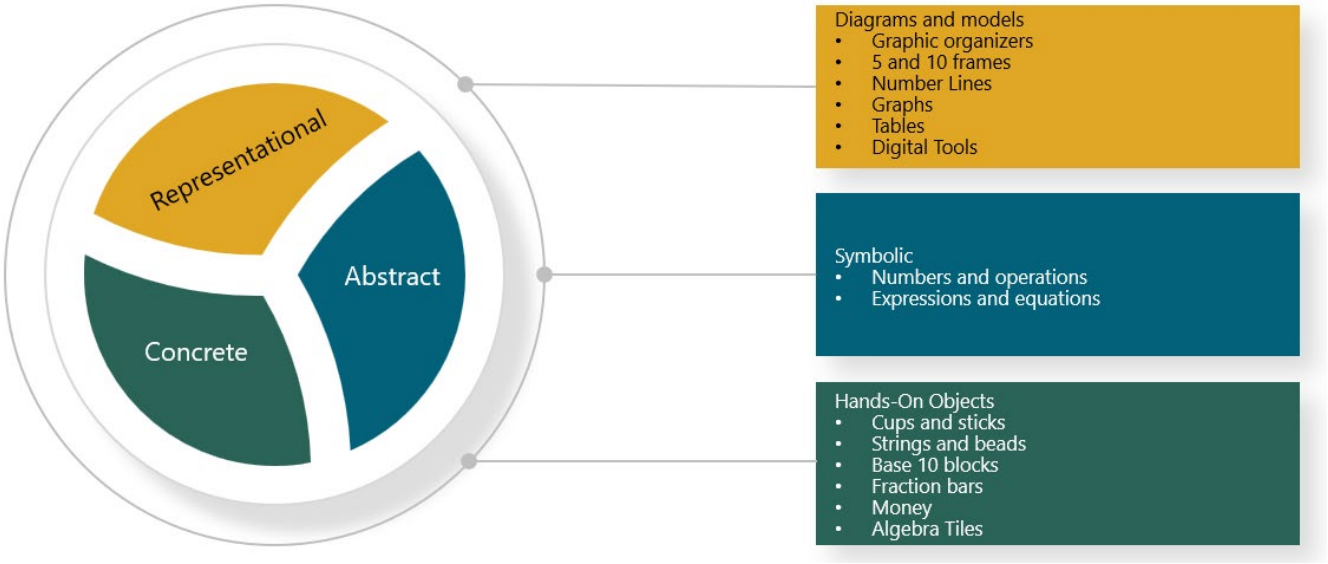


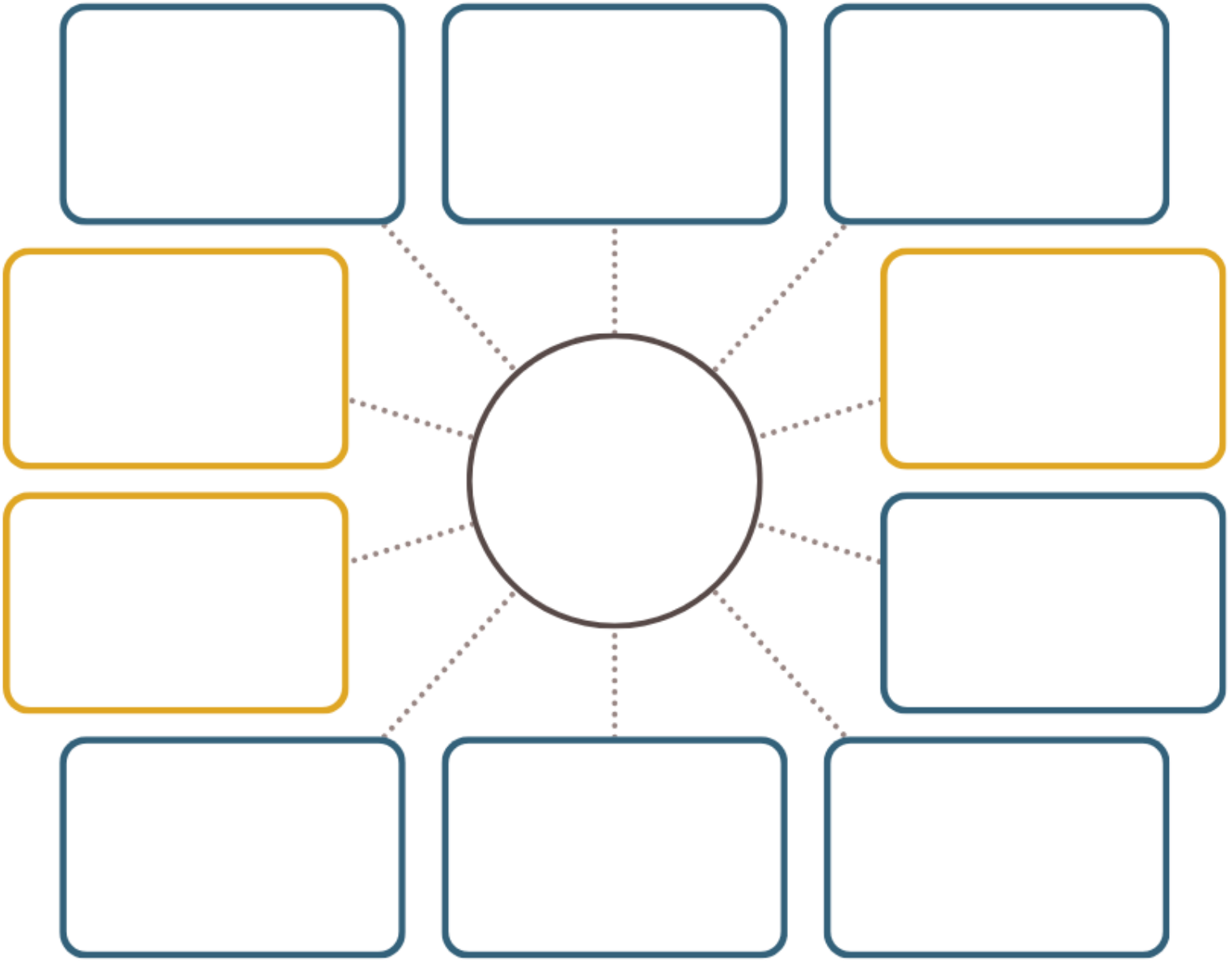
Figure 13: Concrete-Representational-Abstract

Textbook Look Through Recording Sheet

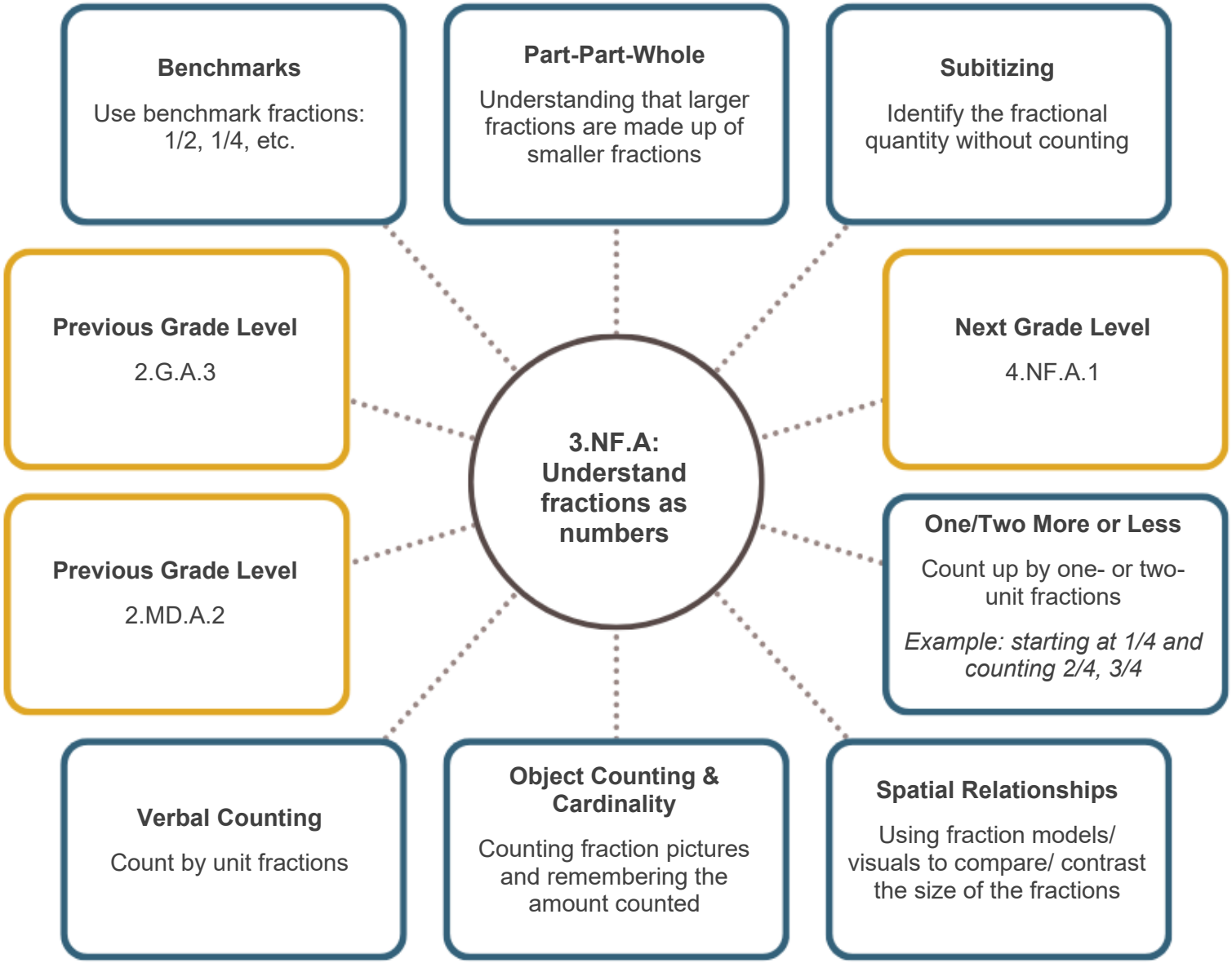
Concept	Found?	Where in the lesson?	Notes/Gaps
Subitizing			
Verbal Counting			
Object Counting			
Cardinality			
Spatial Relationships			
One/Two More & Less			
Benchmark Numbers			
Part-Part-Whole			
Concrete			
Representational			
Abstract			

What's Missing	Where I could bring it in	How (Specific Action)

Blank Web (Module 3 Immersion Experience)



Example Web Diagram (Module 3 Immersion Experience)



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