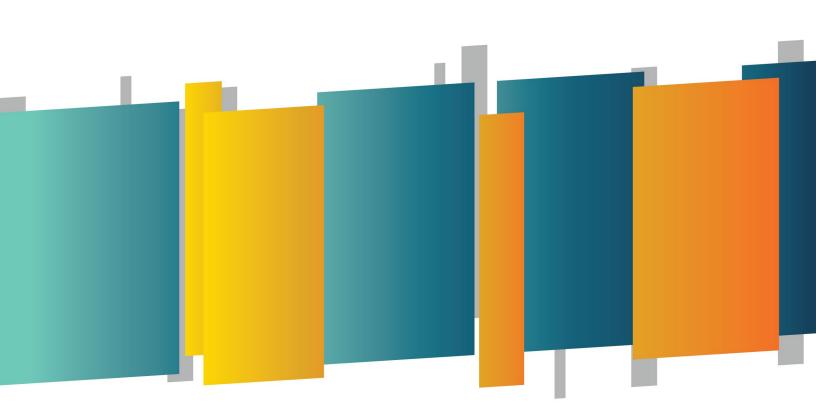
# **Mathematics Teacher Preparation Workbook**



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#### Introduction

#### Context

The Department's mission is to ensure all students experience a world-class education. This mission aims to "Equip lowa's future workforce with a strong foundation in mathematics by increasing the percentage of students who demonstrate mathematical proficiency and problem-solving ability."

The Iowa Mathematics Educator Preparation Workbook (Workbook) evolved from the requirements for educator preparation detailed in Iowa Administrative Rules and the legislative expectations outlined in <u>House File 784 (HF 784)</u>. This legislation emphasizes the importance of structured, evidence-based mathematics instruction, early identification of student needs, and high-quality instructional materials aligned with the <u>Iowa Academic Standards for Mathematics</u>.

The competencies to effectively teach mathematics in Iowa were derived from the following foundational resources:

- <u>lowa Comprehensive State Mathematics Plan</u>
- Iowa Academic Standards for Mathematics
- National Council of Teachers of Mathematics (NCTM) Principles to Actions
- NCTM's Principles, Standards, and Expectations
- The Mathematical Education of Teachers (MET) II
- Association of Mathematics Teacher Educators (AMTE) Standards for Preparing Teachers of Mathematics

This Workbook provides guidance and examples of content for educator preparation programs, including ensuring teacher candidates are equipped with the knowledge, skills, and dispositions necessary to deliver high-quality mathematics instruction. The Workbook offers program-level tools for evaluation, curriculum mapping, and reflection.

- Appendix 1: Program Self-Analysis
- Appendix 2: Curriculum Map
- Appendix 3: Resources for High-Quality Mathematics Teacher Preparation
- Appendix 4: Principles, Standards, and Expectations

As mathematics education evolves, educator preparation programs must maintain mechanisms to stay current with emerging research and instructional practices. Programs are encouraged to regularly review and refine their coursework, methods, and clinical experiences to ensure that candidates are prepared to meet the diverse needs of lowa's learners and implement rigorous, equitable, and evidence-based mathematics instruction.

#### **Alignment and Collaboration**

This Workbook aligns with HF 784, lowa's educator preparation requirements, the <a href="Lowa Comprehensive State">Lowa Comprehensive State</a> Mathematics Plan, the <a href="Lowa Academic Standards for Mathematics">Lowa Academic Standards for Mathematics</a>, and national mathematics education frameworks, including the <a href="NCTM">NCTM's Principles</a>, Standards, and Expectations, <a href="The The Mathematical Education of Teachers (MET) II</a> and the <a href="AMTE Standards for Preparing Teachers of Mathematics">AMTE Standards for Preparing Teachers of Mathematics</a>. The Workbook supports the implementation of the educator preparation requirements defined in HF 784, including integrating mathematics methods coursework and developing teacher candidate competencies in number sense, learning progressions, conceptual understanding, procedural fluency, and application. Additionally, it facilitates the implementation of high-quality instructional materials aligned to lowa academic standards for mathematics.

This collaborative approach ensures that the Workbook reflects Iowa's collective commitment to preparing confident, capable, and well-supported mathematics educators who are equipped to deliver high-quality, standards-aligned instruction to all learners.

#### **Purpose**

This Workbook supports lowa's commitment to high-quality mathematics instruction by outlining clear expectations for teacher preparation programs. It provides tools to ensure teacher candidates gain the knowledge and skills needed to deliver standards-aligned, evidence-based mathematics instruction that meets the diverse needs of all students.

Aligned with the Principles and Standards for School Mathematics developed by the National Council of Teachers of Mathematics (NCTM), Appendix 4 emphasizes the conditions necessary for effective teaching and learning. Grounded in six foundational principles—Equity, Curriculum, Teaching, Learning, Assessment, and Technology—and supported by both Content and Process Standards, these national guidelines define the essential knowledge, skills, and dispositions students should develop from prekindergarten through grade 12. They provide a coherent framework to guide teacher preparation and ensure all students experience meaningful, rigorous, and equitable mathematics learning.

## Part 1: Iowa Mathematics Educator Competencies

#### **Domain 1: Understanding Mathematical Development**

A mathematics educator with a comprehensive understanding of mathematical development can support students in constructing knowledge through meaningful, developmentally appropriate instruction. Drawing on research-based learning progressions and frameworks, the educator recognizes that students gradually build understanding through promoting student-centered learning. A knowledgeable educator demonstrates their knowledge through the following competencies:

- 1.1 Reasonable goals and expectations for learners at various stages of mathematics development, including familiarity with the Iowa Academic Standards for Mathematics and the structure of high-quality instructional materials aligned to those standards.
- 1.2 Understanding how students construct mathematical knowledge through the development of conceptual understanding, procedural fluency, and application, emphasizing fluency through strategic reasoning and metacognition rather than speed or memorization alone.
- 1.3 Demonstrates awareness of key learning progressions:

1.3.a In K-6, that builds number sense, including:

- Subitizing
- Cardinality
- Object counting
- Verbal counting
- Spatial relationships and use of
- Benchmark numbers (e.g., 5 and 10)
- Part–part–whole reasoning

#### **Early Numeracy Progression**

Domain	Progression	Description
Subitizing	Perceptual → Conceptual	Children instantly recognize small quantities and then begin to understand how parts compose a whole.
Cardinality	Counting to Find Out "How Many" → Understanding the Last Number Represents the Total	Children realize that the last number word said in a count tells "how many" are in the entire set.
Object Counting	One-to-One Correspondence → Keeping Track Accurately	Children learn to match each object with one number word and develop strategies to count systematically.
Verbal Counting	Emerging Number Word Sequence → Stable Order	Children begin by informally reciting number words and gradually develop accuracy and consistency in counting.
Spatial Relationships	Relative Position → Mental Mapping	Children learn positional words (e.g., "next to," "under") and progress to visualizing and manipulating shapes and numbers in space.
Benchmark Numbers	Familiarity with 5 and 10 → Use in Composition	Children recognize 5 and 10 as key anchors and use them to efficiently combine and break apart numbers.
Part–Part–Whole	Recognizing Parts → Understanding Number Structure	Children begin to see numbers as composed of two or more parts and use this understanding to solve problems.

#### **Secondary Number and Algebra Progression**

Domain	Progression	Description
Rational Numbers	Concrete Understanding → Abstract Reasoning	Students move from manipulating rational numbers in concrete contexts (e.g., visual models, number lines) to applying operations with flexibility and efficiency.
Proportional Relationships	Multiplicative Comparison → Functional Reasoning	Students progress from comparing ratios and rates to representing proportional relationships as linear functions and interpreting slope and rate of change.
Expressions and Equations	Numerical Patterns → Symbolic Manipulation	Students advance from identifying numerical patterns to writing, simplifying, and solving increasingly complex expressions and equations symbolically.
Functions	Pattern Recognition → Modeling and Analysis	Students develop from recognizing patterns in input- output tables to interpreting, analyzing, and modeling relationships using function notation and graphs.
Structures of the Real Number System	Operational Properties → Understanding of Number Systems and Closure	Students move from using properties like commutativity and distributivity to understanding the structure, hierarchy, and closure of real and complex number systems.

#### Domain 2: Instructional Practices for Diverse Learners - "The Who"

An effective mathematics educator designs and delivers instruction that is responsive to learners' diverse strengths, needs, and backgrounds. This includes recognizing the variability in how students progress along learning progressions and ensuring all students, especially those struggling, have access to meaningful, grade-level mathematics. A knowledgeable educator demonstrates their understanding through the following competencies:

- 2.1 Understands and responds to students' diverse cultural backgrounds, languages, identities, learner differences, and lived experiences that shape their mathematical thinking.
- 2.2 Integrates developmentally appropriate, evidence-based supports for students with disabilities and those identified as needing intervention in mathematics.
- 2.3 Demonstrates a commitment to high expectations for all students.

#### **Domain 3: Structured Mathematics Instruction – "The How"**

An effective mathematics educator engages students in learning experiences that are intentional, responsive, and grounded in how students learn mathematics. Instruction is purposefully designed—leveraging high-quality instructional materials (HQIM), sequencing tasks, and providing support that guides students from informal strategies to more abstract and efficient reasoning. A knowledgeable educator demonstrates their understanding of high-quality mathematics instruction, as outlined in Principles to Actions: Ensuring Mathematical Success for All (NCTM, 2014), through the following competencies:

3.1 Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates these goals within learning progressions, and uses them to guide instructional decisions.

- 3.2 Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem-solving, allowing multiple entry points and varied solution strategies.
- 3.3 Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematical concepts and procedures, and as tools for problem solving.
- 3.4 Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students, enabling them to build a shared understanding of mathematical ideas by analyzing and comparing their approaches and arguments.
- 3.5 Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense-making about important mathematical ideas and relationships.
- 3.6 Build procedural fluency from conceptual understanding. Effective teaching of mathematics fosters fluency with procedures on a foundation of conceptual understanding, enabling students to become skillful in using procedures flexibly as they solve contextual and mathematical problems over time.
- 3.7 Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
- 3.8 Elicit and use evidence of student thinking. Effective teaching of mathematics utilizes evidence of student thinking to assess progress toward mathematical understanding and continually adjust instruction in ways that support and extend learning.

#### **Domain 4: Structured Mathematics Instruction – "The What"**

An effective mathematics educator possesses deep knowledge of essential mathematics content and how students learn that content over time. High-quality instruction connects students' informal understandings to formal mathematical concepts through developmentally appropriate, intentional teaching. Drawing on learning and developmental progressions, student reasoning, and conceptual frameworks, educators foster procedural fluency through experience rather than memorization or repetitive practice. Skilled mathematics educators demonstrate the following:

- 4.1 Uses learning and developmental progressions across K–12 to design instruction that is responsive to how students build an understanding of specific mathematical ideas. This includes foundational concepts such as number sense and operations in early grades, as well as algebraic thinking, functions, and modeling in secondary grades (e.g., Carpenter, Clements & Sarama, Van de Walle, NCTM, AMTE).
- 4.2 Builds procedural fluency through flexible strategy use, guiding students to select, apply, and justify efficient strategies after developing conceptual understanding, prioritizing reasoning over memorization.
- 4.3 Anticipates and addresses common misconceptions by analyzing students' reasoning to identify developmental stages and inform instructional decisions that support learning.

#### **Domain 5: Assessment Practices in Mathematics**

An effective mathematics educator uses assessment intentionally and equitably to inform instruction, monitor student growth, and support access to grade-level mathematics. This includes selecting appropriate tools, interpreting data within learning progressions, and collaborating to meet student needs. A knowledgeable educator demonstrates their understanding through the following competencies:

- 5.1 Selects and appropriately uses a range of assessment types, such as formative and summative, to support instructional decisions and student learning.
- 5.2 Analyzes student work and performance-based assessments to determine appropriate next steps—providing feedback that supports conceptual development and extends learning.
- 5.3 Supports student progress within a Multi-Tiered System of Supports (MTSS) framework using assessment data to inform instructional goals.

#### Part 2: Process for Evaluation

#### **Evaluation Criteria: Key Performance Categories**

Preservice and practicing teachers must understand that mathematical proficiency is not innate but developed through intentional, coherent instruction grounded in how students learn. Given the impact of mathematics on lifelong success and full participation in society, it is imperative that Iowa Educator Preparation Programs (EPPs) rigorously design and evaluate their coursework to ensure alignment with evidence-based mathematics instruction. To support this goal, Iowa's EPPs can engage in a process to determine how well their courses align with essential instructional principles in mathematics.

When assigning a rating of meets, developing, or does not meet, each competency must demonstrate evidence aligned to benchmark criteria in the following three key performance categories:

- a. Materials/Curriculum
- b. Assessments to Demonstrate Content Knowledge
- c. Practical Application

#### (a) Materials/Curriculum

The materials (e.g., textbooks, readings, podcasts, slide presentations, and other resources) must align with evidence-based mathematics instruction grounded in learning progressions, cognitive research, and effective teaching practices.

To meet the benchmark, EPPs will:

Use resources aligned with evidence-based mathematics instruction.

(Programs should provide a rationale for the use of materials that are not aligned with widely accepted resources, such as those endorsed by organizations like the National Council of Teachers of Mathematics (NCTM), the Association of Mathematics Teacher Educators (AMTE), or reviewed by EdReports.)

Benchmark	Does Not Meet (0)	Developing (1)	Meets (2)
(a) Materials/Curriculum	Textbooks and supporting readings/videos do not meet expectations.	Textbook and supporting readings/videos rationale is unclear, or one text is not aligned.	Textbooks and supporting readings/videos provide clear alignment with accepted resources.

#### (b) Assessments to Demonstrate Content Knowledge

EPPs design learning opportunities with clear outcomes aligned with fair, reliable, valid, and rigorous assessments administered in the program.

To meet the benchmark, EPPs will:

 Provide clear evidence that candidates have mastered the lowa Mathematics Educator Competencies through meaningful measures, excluding candidate self-reflection and classroom discussion as sole sources of evidence.

Benchmark	Does Not Meet (0)	Developing (1)	Meets (2)
(b) Assessments to Demonstrate Content Knowledge	No graded written work, tests, or quizzes.  Programs may expose candidates to the criteria, but evidence is limited of candidate performance.	Assessment rigor is unclear; it is difficult to determine if the content knowledge of each candidate is measurable through the practices included in the course.	Includes quality, rigorous evaluations of learning throughout the course and well-planned end-of-course assessments. Candidates are given clear opportunities to demonstrate content knowledge, and practices are in place for reviewing material that is not understood.

#### (c) Practical Application

Candidates demonstrate their ability to implement mathematics competencies and reflect on their learning in a practicum.

To meet the benchmark, EPPs will:

- Explicitly connect clinical elements to the Iowa Mathematics Educator Competencies.
- Design opportunities for candidates to implement evidence-based instructional programs prior to student teaching.

Benchmark	Does Not Meet (0)	Developing (1)	Meets (2)
(c) Practical Application	Clinical opportunity is not clear or may be observation-based.	Clinical opportunity is generally tied to competencies.	Clinical opportunity is reflectively and explicitly tied to the five overarching competencies. Clinical experience is embedded in the program prior to student teaching. Clinical experience is embedded in the program prior to student teaching.

#### Part 3: Evaluation of Mathematics Curricula

By Fall of 2025, the preparation programs are expected to review the mathematics curriculum, incorporating aggregate data related to candidates' performance on content assessments and coursework artifacts. Starting in Fall 2025, it is expected that programs ensure alignment with the Chapter 13 standards and the Iowa Comprehensive State Mathematics Plan. To support this process, the Department is providing the Program Self-Analysis Tool in Appendix 1 for institutions to conduct a self-study. Starting in Fall 2025, programs are expected to demonstrate benchmark proficiency aligned to each of the Iowa Mathematics Educator Competencies during the scheduled program review. Other curriculum reviews may be expected outside of the regularly scheduled site visits.

#### **Program Self-Analysis and Curriculum Improvement**

Programs may use the Program Self-Analysis Tool (Appendix 1) to collect evidence showcasing how the program meets each competency. As described above in Part 2: Process for Evaluation, this includes demonstrating:

- a. The materials and curriculum that the program is using.
- b. The assessments used to demonstrate that each candidate has acquired the content knowledge.
- c. The practicum experiences aligned with the Iowa Mathematics Educator Competencies in which candidates will engage.

#### **Review and Feedback for Program Improvement**

To provide a deeper evaluation to support programs in curricular revision (if needed), the Department consultants will offer office hours to support these curricular improvements.

When reviewing preparation programs, the Educator Quality team and other department consultants will use the same self-assessment tool that is provided in the Appendix 1 Program Self-Analysis Tool, including artifacts (such as syllabi and sample work).

Preparation programs are encouraged to submit the most updated curriculum (if changes are substantial) to the Board of Educational Examiner (BoEE), through the regular curricular submission process prior to the 2025-2026 academic year. Upon the regular review of the approved curricula, the preparation programs may be notified and required to update additional evidence within their approved curriculum to BoEE.

## Appendix 1: Program Self-Analysis/Phase 1

Programs may use the <u>Program Self-Analysis Tool</u> to document alignment with the Mathematics Educator Competencies, referencing the performance indicators outlined in Part 2 of this workbook.

Programs use this tool to provide justification with detailed syllabi, assignments, student work, and coursework artifacts. Programs must ensure that evidence is not simply criteria-based (e.g., exposure to concepts), but performance-based (e.g., demonstration that candidates meet the competencies through practice, application, and assessment). Evidence may include course assignments, observation rubrics, student work analysis, or practicum evaluations.

To support coherence across the preparation program, programs should also consider the scope and sequence of mathematical ideas and instructional practices, identifying when a concept or skill is first introduced, when it is reinforced, and when candidates are expected to demonstrate mastery. For example, an understanding of learning trajectories might be introduced in an early childhood methods course, reinforced through a developmental psychology course, and mastered in a final practicum with application in lesson planning and responsive instruction. Explicit attention to this progression will help ensure that candidate learning builds over time and aligns with the developmental nature of mathematics teaching.

## **Appendix 2: Curriculum Map**

Programs may utilize the <u>Curricular Mapping Tool</u> to demonstrate how mathematics content and competencies are introduced, taught, and assessed across coursework and field experiences. This includes alignment to HF 784, the Iowa Academic Standards for Mathematics, and research-based developmental frameworks such as learning progressions and strategy-based fluency instruction. A collaborative curriculum mapping process, grounded in the Iowa Mathematics Educator Competencies, can be a powerful tool for reflection, helping programs ensure coherence, identify gaps, and develop performance-based, developmentally appropriate mathematics instruction.

## Appendix 3: Resources for High-Quality Mathematics Teacher Preparation

After completing a self-evaluation using this Workbook, educator preparation programs can use the following curated resources to strengthen coursework, clinical experiences, and syllabi. These tools align with the instructional expectations of House File 784 and support the preparation of teacher candidates to deliver standards-aligned, equity-focused, and effective mathematics instruction across grades K–12.

- 1. Iowa Statewide Documents for Mathematics Instruction
  - a. House File 784
  - b. <u>Iowa Academic Standards for Mathematics</u>
  - c. <u>Iowa Comprehensive Statewide Plan</u>
    - i. Executive Summary
  - d. Model Personalized Plan
  - e. Approved Statewide Assessment
  - f. Mathematics Professional Development for K-6 Educators
  - g. Multi-Tier System of Supports
- 2. High Quality Instructional Materials
  - a. EdReports.org Reviews of High-Quality Math Instructional Materials
  - b. Illustrative Mathematics Content Resources and Standards of Mathematical Practices
  - c. Student Achievement Partners Math Language Routines and Task Analysis Tools
- 3. Progressions
  - a. Mathematics Professional Development for K-6 Educators

- b. Progressions for State Mathematics Standards
- c. Progression Videos
- d. Cognitive-Based Approach
- e. Coherence Map
- f. Learning and Teaching with Learning Trajectories
- g. Development and Research in Early Mathematics Education (DREME)
- h. OGAP
- 4. Effective Teaching Practices
  - a. NCTM's Principles to Actions: Ensuring Mathematical Success for All
  - b. What Works Clearinghouse: Practice Guides for Mathematics Instruction
  - c. AMTE Standards for Preparing Teachers of Mathematics
- 5. Fluency
  - a. Figuring Out Fluency Going Beyond Basic Facts and the Companion Site
  - b. Figuring Out Fluency in Math: Because fluency practice is not a worksheet
  - c. National Research Council: Adding It Up Helping Children Learn Mathematics

## Appendix 4: Principles, Standards, and Expectations

The NCTM Principles and Standards for School Mathematics provide a comprehensive framework to guide high-quality mathematics education. Grounded in six foundational principles—Equity, Curriculum, Teaching, Learning, Assessment, and Technology—the document outlines essential conditions for effective mathematics instruction. It also articulates five Content Standards and five Process Standards that define the mathematical knowledge, skills, and habits of mind students should develop from prekindergarten through grade 12, ensuring coherent, equitable, and meaningful learning experiences.

Educational decisions made by teachers, school administrators, and other professionals have important consequences for students and society. The Principles for school mathematics provide guidance in making these decisions.

The six Principles address overarching themes:

- Equity. Excellence in mathematics education requires equity—high expectations and strong support for all students.
- Curriculum. A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well-articulated across the grades
- Teaching. Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- Learning. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- Assessment. Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
- Technology. Technology is essential in teaching and learning mathematics, as it influences the mathematics taught and enhances students' learning.

The Standards for school mathematics describe the mathematical understanding, knowledge, and skills that students should acquire from prekindergarten through grade 12. Each Standard consists of two to four specific goals that apply across all the grades.

The five Content Standards each encompass specific expectations, organized by grade bands:

- Number & Operations
- Algebra

- Geometry
- Measurement
- Data Analysis & Probability

The five Process Standards are described through examples that demonstrate what each standard looks like and what the teacher's role is in achieving it:

- Problem Solving
- Reasoning & Proof
- Communication
- Connections
- Representation
- Bundled Content Standards by Grade Band

The <u>Principles</u>, <u>Standards and Expectations document</u> organizes the mathematics content standards into the linked grade band bundles—K–8, 5–12, and K–12—to support coherence and progression across educator preparation programs. By viewing the standards through these grouped lenses, programs can ensure candidates understand how mathematical content develops over time and how to scaffold instruction that builds on students' prior knowledge while anticipating future learning. Programs use these bundled standards to inform curriculum design, shape clinical experiences, and support instructional planning across grade spans.

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